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Three Dimensional Trajectory Planning of Unmanned Aerial Vehicles Based on Quantum Differential Search

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Abstract: In this paper, an improved differential search (DS) algorithm was proposed to solve the trajectory planning problem. The DS algorithm is a new kind of evolutionary algorithm that developes rapidly recently. However, slow convergence speed and easily trapping into local optimum of the DS algorithm are the main disadvantages that limit its further application. To overcome the disadvantages of DS algorithm, we propose a quantum differential search (QDS) algorithm for solving the three dimensional trajectory planning of unmanned aerial vehicles (UAVs) in this paper. It is a combination of quantum theory and principles of differential search algorithm. By using the quantum strategy, the improved DS algorithm can escape the local optimum. The non-winner particles are mutated by quantum theory and the global best position is mutated by using the small extent of disturbance according to the variance ratio of fitness. Series of experimental results demonstrate the feasibility, effectiveness and robustness of our proposed method. The results show that the proposed QDS algorithm can effectively improve both the global searching ability and the speed of convergence.

Keywords: Unmanned aerial vehicles (UAVs), differential search (DS), quantum, trajectory planning.

1 Introduction

The demands for a reliable and cost-effective access to space for military applications have sparked a renewed interest in unmanned aerial vehicles (UAVs). Research on UAVs can directly affect battle effectiveness of the air force, therefore is crucial to safeness of a nation. UAV mission planning is particularly significant, and trajectory planning for UAVs is one of the most important issues of mission planning. Trajectory planning is to generate a trajectory between an initial prescribed location and a desired destination having an optimal or near optimal performance under specific constraints [1]-[8]. In recent years, many computing methods have been used to UAVs mission assignment and trajectory planning, such as genetic algorithm (GA) [9], artificial physics [10]. Differential search (DS) algorithm is a new evolutionary computation technique, which owns the advantages of simple implementation, a good optimization result and short computation time. Therefore, an improved DS algorithm is formulated for UAVs trajectory planning in this paper.

DS algorithm is an algorithm developed for solution of optimization problems [11]. The DS algorithm simulates the Brownian-like random-walk movement used by an organism to migrate. The convergence speed and the stability have exceeded other stochastic algorithm, like annealed nelder and mead (ANM) strategy, adaptive simulated annealing (ASA), and evolution strategies (ES). However, it can easily trap into the local best, and the algorithm may end up without finding a satisfying trajectory. Considering the outstanding performance of quantum theory in jumping out of stagnation, we introduced it to improve the robustness of the basic DS algorithm, and the comparative experimental results testified that our proposed method manifests better performance than the original DS algorithm.

The rest of this paper is organized as follows. Section 2 introduced the environment modeling for UAV trajectory planning. Subsequently, Section 3 described the principles of the basic DS algorithm, The trajectory smoothing method is given in the Section 4, and Section 5 specified implementation procedure of our proposed the quantum differential search (QDS) algorithm . Then, in Section 6, series of comparison experiments are conducted. Our concluding remarks are contained in the final section.

2 Environment Modeling for UAV trajectory Planning

Modeling of the threat sources is the key task in UAV optimal trajectory planning. There are two types of threat sources: artificial threats and natural threats. The artificial threats include the enemy's radar, missiles and artillery and so on [13]. We can choose appropriate models of them under different circumstances. In this work, we use the circle model to describe the threat sources, and the radius of the circle is the range of threat source. We also define the treat level for calculating the threat costs. Mathematically, the problem of trajectory planning for UAVs can be described as follows :

Given the launching site A and target site B, ($A,B \in 3R$), K threat sets $\{T_1, T_2, \ldots\ldots, T_k\}$, and the parameters of UAV's maneuvering performance constraints (such as the restrictions of turning angle α, climbing/diving angle β,and flying height h, etc.), find a set of waypoints $(W_0, W_1, \ldots\ldots, W_n, W_{n+1})$ with $W_0 = A$ and $W_{n+1} = B$ such that the resultant trajectory is safe and flyable. In other words , for the reported trajectory, no line segment intersects

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the interior of any $T_1, T_2, \ldots, T_k$ and all constraints are satisfied *B*.

Suppose that the terrain of the environment and the information of threat regions are known, and the start and aim points are also given. We may obtain a high-quality flight trajectory between the start and aim points for UAVs, and the cost function of flight air trajectory can be defined as follows[14].

$$F = W_1 f(l) + W_2 f(h) + W_3 f(c) + W_4 f(d) + W_5 f(f) \quad (1)$$

where W_1, W_2, W_3, W_4 and W_5 are weight coefficients, and they satisfy $W_1 + W_2 + W_3 + W_4 + W_5 = 1$.

For the given trajectory, the length cost *f*(*l*) is defined as:

$$f(l) = \sum_{i=1}^{n} l_i^2 \quad (2)$$

where *i* is the length of the *i*th trajectory segment, and we suppose that the length of the *i*th straight line are the actual length of the trajectory.

The height cost *f*(*h*) is defined as

$$f(h) = \frac{A_{traj} - Z_{\min}}{Z_{\max} - Z_{\min}} \quad (3)$$

where $Z_{\max}$ is the upper limit of the elevation in our search space, $Z_{\min}$ is the lower limit, and A_{traj} is the average altitude of the actual trajectory. $Z_{\max}$ and $Z_{\min}$ are respectively set to be slightly above the highest and lowest points of the terrain.[15]

The term associated with ground collisions is defined as follows

$$f(c) = \begin{cases} 0, & L_{under} = 0 \\ p + \left(\dfrac{L_{under}}{L_{traj}} \right), & L_{under} > 0 \end{cases} \quad (4)$$

where L_{under} is the total length of the subsections of the trajectory which travels below the ground level and L_{traj} is the total length of the trajectory. We compare the altitude of the terrain and the altitude of the trajectory in a discrete way using the Bresenham's line drawing algorithm [15]:

The term associated with the violation of the danger zones is defined as follows

$$f(d) = \frac{L_{insided\,d,z}}{\sum_{i=1}^{n} d_i} \quad (5)$$

where *n* is the total number of danger zones, is $L_{insided\,d,z}$ the total length of the subsections of the trajectory which go through danger zones and d_i is the diameter of the danger zone *i*. This definition ensures that a trajectory passing through a single danger zone is severely penalized on a map containing few danger zones and lightly penalized on a map containing many danger zones. Since it is possible for $L_{insided\,d,z}$ to be larger than $\sum_{i=1}^{n} d_i$ (as in the case of a dog-leg path through a single danger zone), we arbitrarily bound to 1.

The term associated with an insufficient quantity of fuel available is defined as follows

$$f(f) = \begin{cases} 0, & F_{traj} \le F_{init} \\ p + \left(\dfrac{\sum_{i=1}^{n} F_i}{F_{traj}} \right), & F_{traj} > F_{init} \end{cases} \quad (6)$$

where F_i is the quantity of fuel required to fly the imaginary *i*th straight segment point. F_{traj} is the actual amount of fuel needed to fly trajectory, F_{init} is the initial quantity of fuel onboard the UAV.The quantity of fuel required is computed using the approach presented in [15] . By controlling the threat cost defined here, the survival probability of UAVs can be increased efficiently.

3 Principles of the DS Algorithm

In the DS algorithm, artificial-organisms $(i.e, X_i, i = \{1,2,3,\ldots\ldots,N\})$ making up an artificial-organism contain members as much as the size of the problem $(i.e, x_{i,j}, i = \{1,2,3,\ldots\ldots,D\})$. Here, *N* signifies number of elements in the superorganism and *D* indicates size of the respective problem. In the DS algorithm, a member of an artificial-organism in initial position is defined by using low and high limit [11]-[12]. In the DS algorithm, the mechanism of finding a stopover site at the are as remaining between the artificial-organisms maybe described by a Brownian-like random walk model. Randomly selected individuals of the artificial-organisms move towards the targets in order to discover stopover sites, which are very important for a successful migration by randomly change the numbers of the elements. In the DS algorithm, the scale value is produced by using a gamma-random number. The generator controlled by a uniform-random number generator (i.e.,rand) working in the range of [0 1] together. The structure used for calculation of the scale value allows the respective artificial-super-organism to radically change direction in the habitat. In the DS algorithm, a stopover site position is produced by using

$$St = Sp + N \cdot (D - Sp) \quad (7)$$

where *St* is the stopover site, *Sp* is the superorganism, *N* is the scale of the superorganism, and *D* is the superorganism after conducting stochastic mutation.

In the DS algorithm, the members (i.e.individuals) of the artificial organisms of the superorganism to participate in the search process of stopover site are determined by a random process.In the DS algorithm, if one of the elements of stopover site is, for one reason, goes beyond the limits of the habitat (i.e. search space), the said element is randomly deferred to another position in the habitat. If, in the DS algorithm, a stopover site is more fertile than the sources owned by the artificial-organism of which the individuals that discover that *stopover site*, that artificial-organism moves to that stopover site. While the artificial-organisms change site, the superorganism containing the artificial organisms continue sits migration towards the global minimum.

4 Trajectory Smoothing Strategy

Three dimentional trajectory planning is a key issue for aircraft [16, 17]. The main purpose of smoothing the trajectory is using mathematical methods to remove uneven points, and making the search for the optimal trajectory continuous.

Consider the sharp turn trajectory shown in Fig. 1, the trajectory is composed by $\vec{w}_{i-1}$, $\vec{w}_i$ and $\vec{w}_{i+1}$ [18]. Obviously, the trajectory $\overline{w_{i-1}w_i} \rightarrow \overline{w_i w_{i+1}}$ is not available for the UAV flight sections, and it should be smooth.

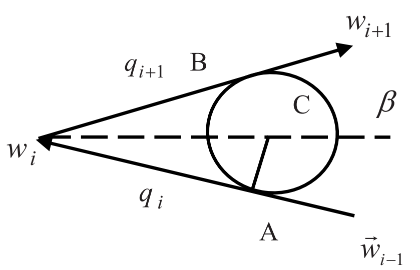

Fig. 1 UAV waypoint smooth

Let $\vec{q}_i$ represent the unit vector from $\vec{w}_{i-1}$ to $\vec{w}_i$ while $\vec{q}_{i+1}$ represent the unit vector from $\vec{w}_i$ to $\vec{w}_{i+1}$, then

$$\vec{q}_i = \frac{\vec{w}_i - \vec{w}_{i-1}}{\left\|\vec{w}_i - \vec{w}_{i-1}\right\|} \tag{8}$$

$$\vec{q}_i = \frac{\vec{w}_{i+1} - \vec{w}_i}{\left\|\vec{w}_{i+1} - \vec{w}_i\right\|} \tag{9}$$

Let β show the vector angle between $\vec{q}_i$ and $\vec{q}_{i+1}$, then, C represent the inscribed circle, whose radius can be expressed as

$$\beta = \arccos(-\vec{q}_{i+1}, \vec{q}_i)$$

$$R = 0.5 \cdot \min\left\{\left\|\vec{w}_i - \vec{w}_{i-1}\right\|, \left\|\vec{w}_{i+1} - \vec{w}_i\right\|\right\} \cdot \tan\frac{\beta}{2} \tag{10}$$

Obviously, the inscribed circle C centered at the corner of the two bisecting lines, therefore, the center position can be expressed as

$$\vec{C}_i = \vec{w}_i + \frac{R}{\sin\frac{\beta}{2}} \cdot \frac{\vec{q}_{i+1} - \vec{q}_i}{\left\|\vec{q}_{i+1} - \vec{q}_i\right\|} \tag{11}$$

Then, after the smoothing process, we can describe the $\overline{w_{i-1}w_i} \rightarrow \overline{w_i w_{i+1}}$ with $\overline{w_{i-1}A} \rightarrow \widehat{A}\widehat{C}\widehat{B} \rightarrow \overline{Bw_{i+1}}$. It can be seen that $\overline{w_{i-1}A} \rightarrow \widehat{A}\widehat{C}\widehat{B} \rightarrow \overline{Bw_{i+1}}$ is a possible trajectory of the UAVs.

5 QDS Algorithm Approach for Trajectory Planning

5.1 The Principles of QDS

In the QDS algorithm, a qubit chromosome as a string of n qubits can be defined as follows

$$q = \begin{bmatrix} \alpha_1, \alpha_2, \cdots, \alpha_m \\ \beta_1, \beta_2, \cdots, \beta_m \end{bmatrix} \tag{12}$$

where $|\alpha_i|^2 + |\beta_i|^2 = 1$,i=1,...,m,m is the number of qubits as well as the string length of the qubit individual. $|\alpha_i|^2$ provides the probability that the qubit will be found in the state of '0', while $|\beta_i|^2$ gives the probability that the qubit will be found in the '1'state. A qubit chromosome is able to represent a linear superposition of all possible solutions. It has a better characteristic of diversity than a chromosome. The process to get a classical chromosome is: bring a random number between 0 and 1, if it is bigger than $|\alpha_i|^2$, this bit in the classical chromosome is '1', else '0' is chosen.

The standard mutation operation is totally random without any directions, and the speed of convergence is slowed down. However, in QEC, the qubit representation can be regarded as a mutation operator.[19]-[20] Directed by the current best individual, quantum mutation is completed through the quantum rotation gate $U(\theta)$, and $[\alpha_i\ \beta_i]^T$ can be updated by the following formula.

$$\begin{bmatrix} \alpha_i' \\ \beta_i' \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ \sin\theta_i & \sin\theta_i \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \tag{13}$$

Fig. 2 describes the polar plot of the rotation operation on a qubit. It explains why the rotation gate can increase the speed of convergence.

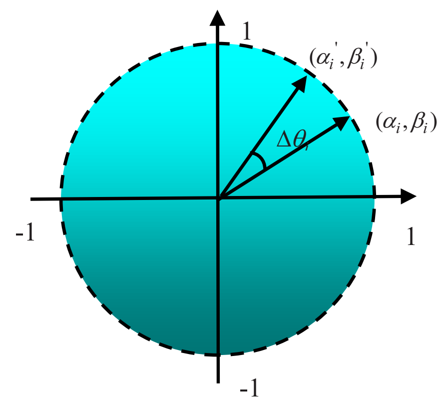

Fig. 2 Polar plot of the rotation gate for qubit chromosome

5.2 Detailed Procedure of Our Proposed Approach

The QDS algorithm is a combination of quantum theory and the basic DS algorithm. The QDS algorithm improved based on quantum theory is superior to the basic DS algorithm theory. The DS algorithm has the ability to converge fast, but sometimes the fast convergence happens in the first a few iterations and relapses into a local optimum easily. By adopting the quantum strategy, we can escape the local optimum as well as increase the speed of reaching the optimal solution. The detailed procedure of our proposed QDS approach to three dimensional trajectory planning of UAVs can be described as follows:

Step1: According to the environmental modeling in section 2, initialize the parameters of D such as solution space dimension D, the population size NP, scaling factor R, the terrain information, the threaten information including the coordinates of threat centers, threat radius and threat levels. Generally, a larger NP will contribute to a larger possibility of finding the best solution of the problem. However, it also means an increased computing complexity of the algorithm. In general, we define NP =(3~10)D. A smaller R will cause premature convergence, while a lager Rwill contribute to improving the capacity of jumping out of the local best. We choose the R=1./*gamrnd* (1,0.5) .(where gamrnd is a function which conducts random matrixs in a special approach)

Step2: Change the original form of the coordinates of the start and aim point, in order to facilitate the calculation of threat consideration. Stochastically generate N trajectories and according to the parameters, calculate the cost of each trajectory formed by relative parameters then we get N feasible solutions based on formulas (1)-(6).

Step3: By the means of the DS algorithm generate new individuals based on formula (7).

Step4: Calculate the fitness of every individual and then compare the cost of the target individuals and the new individuals, select the best individuals into the next generation.

Step5: Conduct the quantum search around the best solution based on formulas (12)-(13) and Fig. 2. Among the engendered series of solutions, select the best one and use it to replace the former best solution.

Step6: Select the best trajectory to record and update the trajectory and make the above individuals after Step 5 into new generation.

Step7: Go to Step 3 until reaching the stopping criteria .

Step8: Change the coordinates to the original form and output the results which were optimized by planning smooth strategy based on formulas (7)-(11).

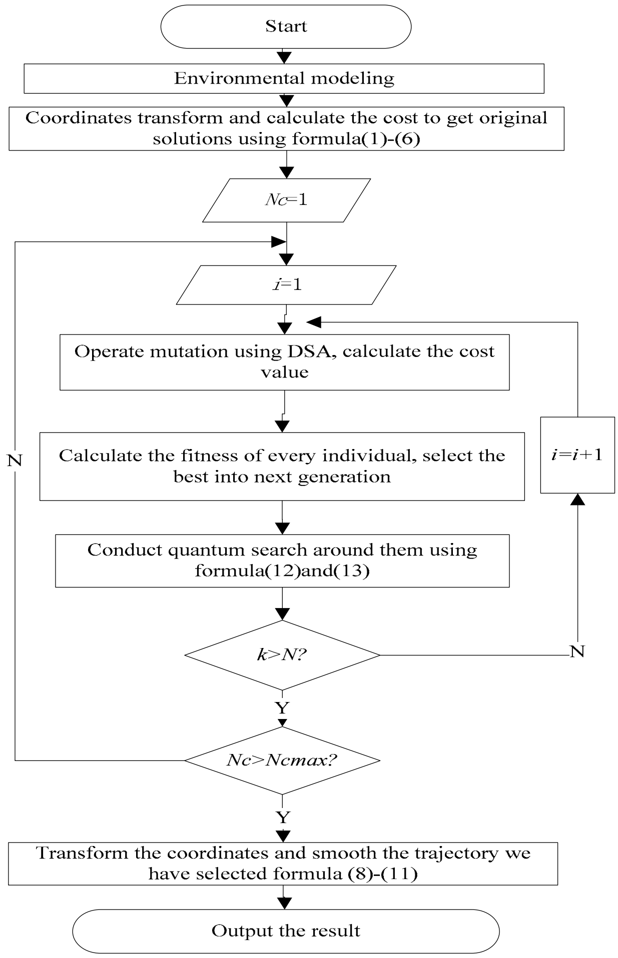

Fig. 3 Flow chart of our proposed QDS for trajectory planning.

6 Experimental Results

In order to investigate the feasibility and effectiveness of our proposed QDS algorithm for UAVs trajectory optimization, a series of comparative experiments have been conducted. The population size is 60 (NP=60). The dimension is 20 (D=15), the maximum iteration number is 200 (Nc_{max}=200) The recommended methods for generation of scale-factor R=1./gamrnd(1,0.5). The comparative results of our proposed QDS with GA, DE and DS algoritms are given in Figs. 4-6.

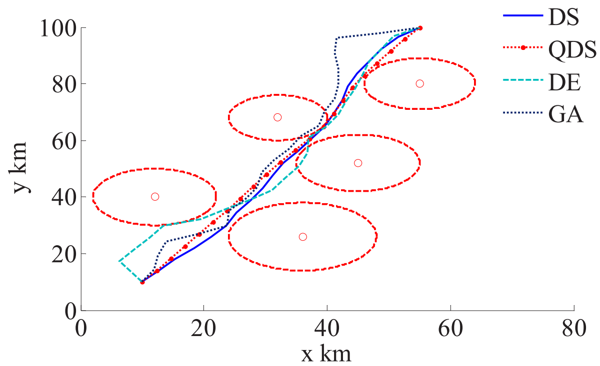

Fig. 4 Comparative trajectory results of QDS, GA, DE and DS

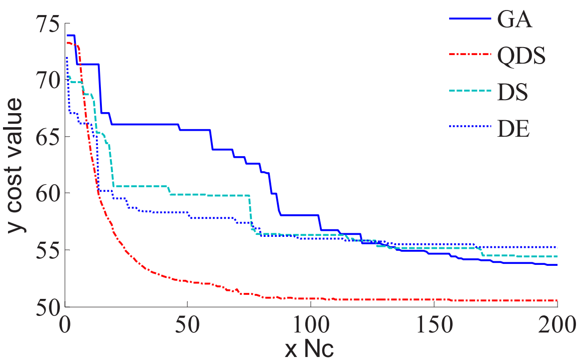

Fig. 5 Comparative curves of QDS, GA, DE and DS

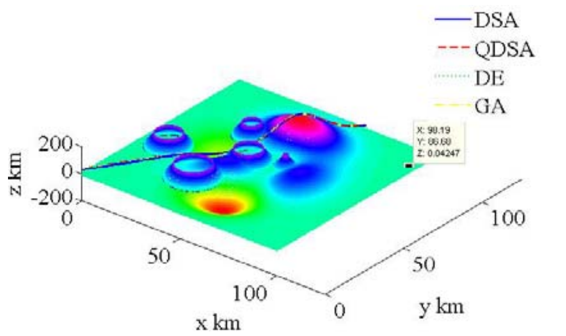

Fig. 6 3D trajectory planning results of QDS, GA, DE and DS

It is obvious that our proposed QDS algorithm can accurately track the given signal faster and more steadily. The trajectory planning results of our QDS algorithm can converge to a smaller range with a faster speed than basic DS, GA and DE algorithms. Our improved algorithm can also jump out of local optimum as well as speeding up the process of finding the optimal solution for three dimentional trajectory planning.

7 Conclusions

In this paper, an improved DS algorithm is proposed for sovling the three dimentional trajectory planning problems of UAVs. Quantum theory is adopted for improving the basic DS algorithm. Comparative experimental results of the proposed the QDS algorithm, basic DS algorithm, GA and DE algorithm are also given to verify the feasibility and effectiveness of our proposed approach, which provide a more effective way for the optimization of flight control parameters. Our future work will focus on applying our proposed the QDS algorithm to the actual flight control system of UAVs.

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