Bloch Quantum-behaved Pigeon-Inspired Optimization for Continuous Optimization Problems*

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Abstract— **In this paper, a novel hybrid Pigeon Inspired Optimization (PIO) and quantum theory is proposed for solving continuous optimization problem. Which is called Bloch Quantum-behaved Pigeon-Inspired Optimization (BQPIO for abbreviation). Quantum theory is adopted to increase the local search capacity as well as the randomness of the position. As a consequence, the improved BQPIO can avoid the premature convergence problem and find the optimal value correctly when solving multimodal problems. An empirical study was carried out to evaluate the performance of the proposed algorithm, which is compared with Particle Swarm Optimization (PSO), basic PIO, and Quantum-behaved Particle Swarm Optimization (QPSO). The comparative results demonstrate that our proposed BQPIO approach is more feasible and effective in solving complex continuous optimization problems compared with other swarm algorithm.**

I. Introduction

Many optimization problems can be solved by population-based swarm intelligence algorithms. The key issue of these meta-heuristic swarm intelligence algorithms are exploration and exploitation as it makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions [1, 2].

Quantum-behaved Particle Swarm Optimization (QPSO) is a modified Particle Swarm Optimization (PSO) which introduce the quantum theory [3]. This algorithm can evade the shortcomings which the standard PSO has. In spite of that, QPSO still have some problems in feasibility and effectiveness to solve some complicated multimodal function. Pigeon-Inspired Optimization (PIO), a population-based swarm intelligence algorithm, is a novel swarm intelligence optimization technique proposed by H. B. Duan in 2014 [4]. It is an effective algorithm, but it will become undependable especially when the population size of the pigeon swarm is small, which I will show in the Section V. To resolve these shortcomings, a new swarm intelligence algorithm with a Bloch Sphere encoding mechanism will be proposed in this paper, which name is Bloch Quantum-behaved Pigeon-Inspired Optimization (BQPIO).

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The remainder of the paper is organized as follows. The next section introduces the main process of PIO. Section III presents the basic mathematical model of BQPIO, and Section IV specifies the detailed implementation procedure of BQPIO. Subsequently, a series of comparison experiments on several benchmark optimization problems are conducted, and the comparative results together with the analysis will be given in Section V. Our concluding remarks and directions for future research are contained in final section.

II. Pigeon-Inspired Optimization

The homing pigeon has an inborn homing ability to find its way home over extremely long distances by using three homing tools: magnetic field, sun and landmarks. Scientists have found that on the top of pigeon's beak, massive iron particles was found which points to the north just as artificial compass. Which helps pigeon to determine its way home. Guilford and his colleagues argue that homing pigeons in different parts of the journey may use different navigation tools. As pigeons start their journey, they may be more dependent on compass-like tools. Whilst in the latter part of their journey, they switch to using landmarks tools when they need to reassess their route [5].

Inspired by the above homing behaviors of pigeons, a novel bio-inspired swarm intelligence optimizer which is named PIO has been invented in 2014 [4]. In order to idealize some of the homing characteristics of pigeons, two operators are designed by using some rules, map and compass operator model is presented based on magnetic field and sun, while landmark operator model is designed based on landmarks.

1) Map and Compass Operator

In the Map and Compass Operator, computer-generated pigeons are used. The position and the velocity of pigeon i can be defined as X_i and V_i, which will update in each iteration in a d-dimension search space. And the new position X_i and velocity V_i of pigeon i at the t-th iteration can be obtained with (1) and (2) [4]:

$$X_i(t+1) = X_i(t) + V_i(t+1) \tag{1}$$

$$V_i(t+1) = V_i(t) \times e^{-Rt} + rand\,(P_g - X_i(t)) \tag{2}$$

where R is the map and compass factor. $rand$ is a random number (generally is a number between 0 and 1). P_g is the global best position of the first t iteration, which can be calculated by comparing all the positions among the whole swarm.

2) Landmark Operator

In the Landmark Operator, we assume the pigeons are still distant from the destination, and obviously they are

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unfamiliar to the landmarks. All pigeons are ranked according their fitness values. Then half of the number of pigeons ($N_p/2$) is decreased according to (3).

$$N_p(t+1) = \frac{N_p(t)}{2} \quad (3)$$

We then find the center pigeon from the left pigeons at the t-th iteration, whose position (X_c) is the desirable destination. This can be described by (4). And the new position of other pigeons can be calculated by (5).

$$X_c(t) = \frac{\sum_{i=1}^{N_p(t)} X_i(t) \times fitness\ (X_i(t))}{N_p(t) \times \sum_{i=1}^{N_p(t)} fitness\ (X_i(t))} \quad (4)$$

$$X_i(t+1) = X_i(t) + rand\,(X_c(t) - X_i(t)) \quad (5)$$

where *fitness*( ) is considered the fitness value of the each pigeon in the swarm. We can choose $fitness(X_i(t)) = \frac{1}{f_{\min}(X_i(t)) + \varepsilon}$ for the minimum optimization problems.

III. Bloch Quantum-Behaved Pigeon-Inspired Optimization

BQPIO is an algorithm combined standard PIO algorithm and a Bloch Sphere encoding mechanism. In this algorithm, every single particle have quantum behavior.

A. Quantum Evolutionary Theory

The probability amplitudes of quantum bit encode the position of particles while the quantum rotation gates perform the changing of the position which achieve particles searching [3, 6, 7], and the quantum bit chromosome can be decreased according to (6) as a string of m quantum bits:

$$q = \begin{bmatrix} \alpha_1 \big| \alpha_2 \big| \alpha_3 \big| \ldots\ldots \alpha_m \\ \beta_1 \big| \beta_2 \big| \beta_3 \big| \ldots\ldots \beta_m \end{bmatrix} \quad (6)$$

where $|\alpha_i|^2 + |\beta_i|^2 = 1$, i=1,...,m. m represents the quantity of quantum bits as well as the string length of the quantum bit individual. $|\alpha_i|^2$ represents the chance that the quantum bit will be found in the ‘0’ state. On the contrary, $|\beta_i|^2$ shows the chance that the quantum bit will be found in the ‘1’ state. The linear superposition of all possible solutions can be represented by quantum bit chromosome and the diversity of it is better than the classic one, then we bring a random number between [0, 1] to get a classical chromosome and compare it with $|\alpha_i|^2$, if it is bigger, this bit in the classical chromosome is 1, otherwise 0 [7-10].

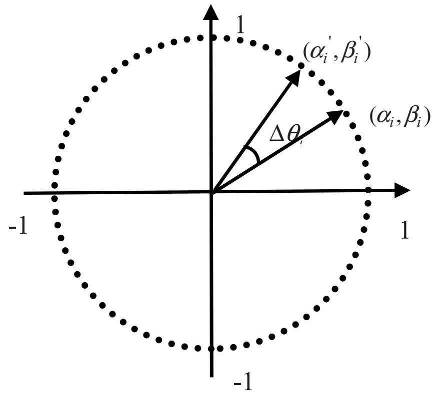

Figure 1. Polar plot of the rotation gate for quantum bit chromosome

The typical mutation operation is completely random without any directions, and the speed of convergence is slowed down. However, the quantum bit representation can be regarded as a mutation operator [3, 8, 11]. Directed by the current best individual, quantum mutation is completed through the quantum rotation gate, and $[\alpha_i\ \beta_i]^T$ can be updated by

$$\begin{bmatrix} \alpha_i' \\ \beta_i' \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ \sin\theta_i & \sin\theta_i \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \quad (7)$$

The polar plot of the rotation gate for quantum bit chromosome can be described as Fig. 1 above. The θ_i can be determined by quantum chromosome and classical chromosome. Which is showed in a table of [9].

B. Bloch Quantum-behaved Pigeon-Inspired Optimization

In order to improve search ability and optimization efficiency and to avoid premature convergence for PIO, a novel algorithm called BQPIO is proposed for continuous space optimization. The classical quantum bits in the Hilbert space’s unit circle have only one variable. So the quantum properties of it are greatly weakened. To solve this problem, PIO is evolved by combining with a Bloch Sphere encoding mechanism.

As shown in Fig. 2, a point P is able to fix by the angle of θ and φ in a situation of 3D Bloch sphere .Every quantum bit has a corresponding point in the Bloch Sphere, hence the particles positions can be directly performed in the Bloch Sphere coordinates. Assuming that $Pigeon_i$ is the i-th pigeon among the swarm. Then the Bloch Sphere encoding process is described as follows [7]:

$$Pigeon_i = \begin{vmatrix} \cos\varphi_{i1}\sin\theta_{i1} \\ \sin\varphi_{i1}\sin\varphi_{i1} \\ \cos\theta_{i1} \end{vmatrix} \begin{matrix} \ldots \\ \ldots \\ \ldots \end{matrix} \begin{vmatrix} \cos\varphi_{id}\sin\theta_{id} \\ \sin\varphi_{id}\sin\varphi_{id} \\ \cos\theta_{id} \end{vmatrix} \quad (8)$$

where $\varphi_{ij} = 2\pi \times rand$, $\theta_{ij} = \pi \times rand$, *rand* is a random number between [0,1]， i = 1, 2, …, N_p; j =1, 2… d; N_p is the population size of the swarm and d is the space dimension.

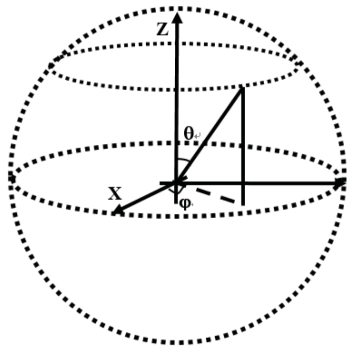

Figure 2. Quantum bit Bloch Sphere

Just like PIO, this algorithm consists of two operators: map and compass operator and landmark operator.

In map and compass operation, the pigeon's position and velocity can be updated according to (9) to (15):

$$m_{best}(t+1) = \frac{1}{N_p}\sum_{i=1}^{N_p} P_i(t) \tag{9}$$

where $m_{best}(t)$ is the average value of the optimal position of each pigeon at the t-th iteration. For each individual pigeon (pigeon i for example), the optimal position of the t-th iteration can be denoted with $P_i(t)$ [12].

We set a 3D mutation operator to show the ability of quantum non-gate in the Hilbert space's unit circle to the Bloch Sphere [7, 10]. The mutation operator satisfies the following matrix (10):

$$B \cdot \begin{bmatrix} \cos\varphi_{ij}(t)\sin\theta_{ij}(t) \\ \sin\varphi_{ij}(t)\sin\varphi_{ij}(t) \\ \cos\theta_{ij}(t) \end{bmatrix} = \begin{bmatrix} \cos(\pi/2-\varphi_{ij}(t))\sin(\pi/2-\theta_{ij}(t)) \\ \sin(\pi/2-\varphi_{ij}(t))\sin(\pi/2-\theta_{ij}(t)) \\ \cos(\pi/2-\theta_{ij}(t)) \end{bmatrix} \tag{10}$$

And the 3D mutation operator we discussed above can be described as (11).

$$B = \begin{bmatrix} 0 & \cot\theta_{ij}(t) & 0 \\ \cot\theta_{ij}(t) & 0 & 0 \\ 0 & 0 & \tan\theta_{ij}(t) \end{bmatrix} \tag{11}$$

Then a new pigeon population was produced.

$$P_iP_g(t+1) = f(t+1)\times P_i(t) + (1-f(t+1))\times P_g(t) \tag{12}$$

$$\omega(t) = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \times \frac{t}{t_{\max}} \tag{13}$$

If

$$f(t+1) \geq 0.5$$

Then

$$X_i(t+1) = P_iP_g(t+1) + \omega(t+1)\times\left|m_{best}(t+1) - X_i(t)\right|\times\ln\frac{1}{f(t+1)} \tag{14}$$

Else

$$X_i(t+1) = P_iP_g(t+1) - \omega(t+1)\times\left|m_{best}(t+1) - X_i(t)\right|\times\ln\frac{1}{u(t+1)} \tag{15}$$

where $P_i(t)$ is the personal best solution found so far by an individual pigeon while $P_g(t)$ represent the current global best position of the first t iteration, which can be calculated by comparing all the positions among the whole swarm. For a single pigeon, the random position between the optimal position of itself and the global best position is $P_iP_g(t)$. $f(t+1)$ *and* $u(t+1)$ are factors of random variables drawn with uniform probability from [0,1], redrawn for each pigeon in every iteration [13]. ω is the constriction factor which can decreases linearly from ω_{max} to ω_{min}, slowing the pigeons as the algorithm is carrying on, so that finer exploration is achieved. In the initial stage, a bigger ω stimulate the search, while in the later period, a smaller ω contribute to the ability of exploitation, so that we can have a high convergence rate [14].

Then we operate the Landmark Operator. The Landmark Operator is just the same as the Landmark Operaor of PIO, half of the number of pigeons is decreased according to (3). We then find the center pigeon of the left pigeons according to (4), the evolution equation can be described as (5).

Finally, we can get the best solution parameters and the best cost value.

IV. Implementation Procedure of BQPIO

The detailed implementation procedure of BQPIO for optimization can be described as follows:

Step 1: Initialize parameters of BQPIO algorithm, such as solution space dimension D, the population size N_p, the map and compass factor R, and the number of iteration t_1 and t_2, where the t_1 is considered the iterations of the map and compass operator, and t_2 will be the total iterations of the whole algorithm.

Step 2: Set each single pigeon with a randomized velocity and position. Then we can calculate the fitness value *fitness ()* of the each pigeon in the swarm.

Step 3: For each individual pigeon, if the particle's current position is better than its own-memory optimal position P_i, then replace the latter with current position. Compare the fitness value of all pigeons, if the current global optimal position is superior to the optimal global position P_g ever searched, then replace the latter with the current global optimal position.

Step 4: Operate the map and compass operator. Firstly, we update the velocity and position of every pigeon according to (9) to (15). Then we compare all the pigeons' fitness value and update the new global best position P_g.

Step 5: If $t > t_1$, stop the map and compass operator and turn to step 6. If not, go to Step 4.

Step 6: Operate the landmark operator. Rank all pigeons according their fitness values, and wave half of pigeons whose fitness value are lower by using (3). We then find the center pigeon of the left pigeons according to (4), which is considered the desirable destination. All pigeons will fly to the destination by adjusting their flying direction according to (5). Next, we can get the best solution parameters and the best cost value.

Step 7: If $t > t_2$, stop the landmark operator, and output the results. Otherwise, go to Step 6.

V. RESULT FROM BENCHMARK SIMULATIONS

In order to investigate the feasibility and effectiveness of the proposed BQPIO, a series of experiments are conducted on several benchmark function problems: Shubert Function, Rosenbrock Function, Rastrigin Function and Schaffer Function.

Shubert Function (f_1) is a complex function of two dimensions, which have 760 extreme points, achieve the minimum at（-1.42513，0.80032）, and can be expressed as follows.

$$\min f(x,y)=\{\sum_{i=1}^{5} i\cos[(i+1)x+i]\}\times\{\sum_{i=1}^{5} i\cos[(i+1)y+i]\} \quad (16)$$

Rosenbrock Function (f_2) is a non-convex function used as a performance test problem, the global minimum, whose value is 0, is inside a long, narrow, parabolic shaped flat valley. To converge to the global minimum, however, is difficult. The function is defined by (17).

$$\min \ f(x_i)=\sum_{i=1}^{D-1}[100\ (x_i^2-x_{i+1}^2)+(x_i-1)^2] \quad (17)$$

Rastrigin Function (f_3) is a non-linear function with multiple peak values. It is quite a difficult problem to find the least value of this function, just because the huge searching space and a mass of local minima. This function can be defined as:

$$\min \ f(x_i)=\sum_{i=1}^{D-1}[x_i^2-10\cos(2\pi x_i)+10] \quad (18)$$

Schaffer Function (f_4) is a two-dimensional complex function, achieve the minimum 0 at（0，0）.

$$\min \ f(x_1,x_2)=0.5+\frac{(\sin\sqrt{x_1^2+x_2^2})^2-0.5}{(1+0.001(x_1^2+x_2^2))^2} \quad (19)$$

The test results and the comparative evolutionary curves of the four functions are showed as TABLE 1, *Fig. 3* to *Fig. 6.*

For the PIO and BQPIO, we set that solution space dimension D is 20, the population size N_p=20, the map and compass factor R=0.2, and the number of iteration t_1 =150, t_2=200.

For the PSO, we set that C_1=C_2=1.4962, ω=0.9, D=20, N_p=20.

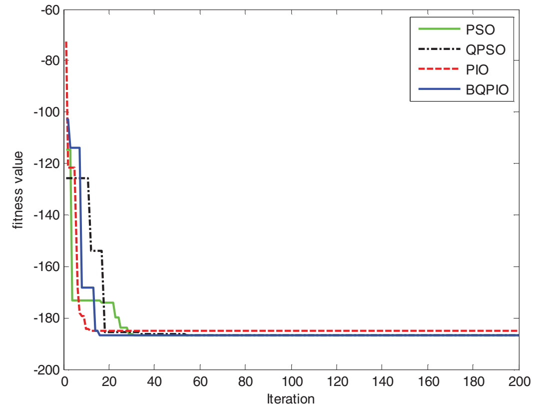

Figure 3. Comparative evolutionary curves of f_1

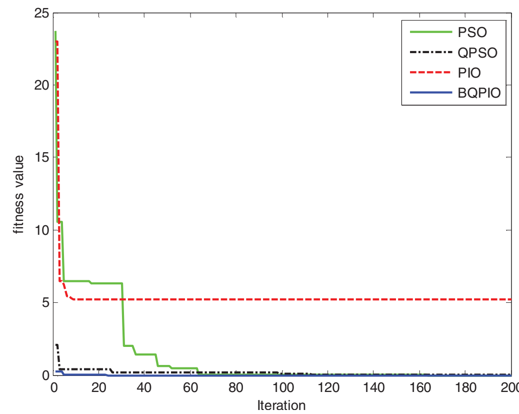

Figure 4. Comparative evolutionary curves of f_2

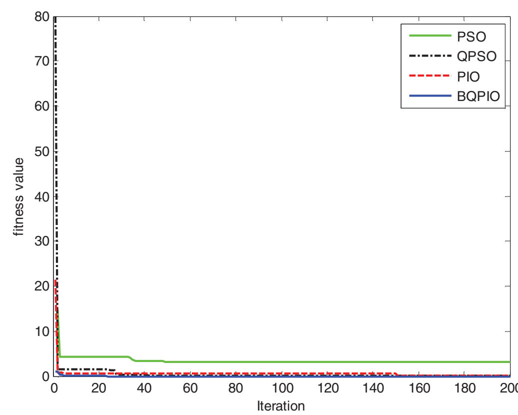

Figure 5. Comparative evolutionary curves of f_3

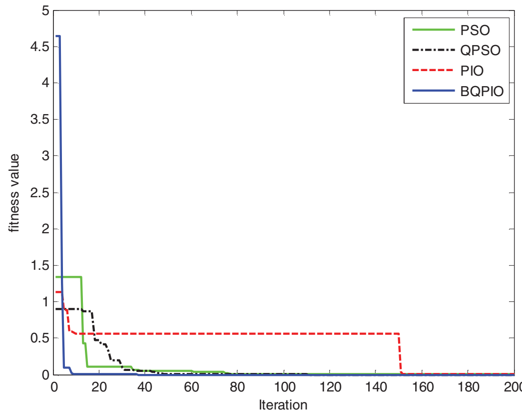

Figure 6. Comparative evolutionary curves of f_4

VI. Conclusion

With the development of technology and the broadening of the scope of engineering problems, the scale and complexity of the problem is increasing. As a result, optimization results of traditional algorithms have many limitations, and the effectiveness of improvements for a definite algorithm is very limited. In this work, we presented a novel hybrid PIO and quantum theory for solving continuous optimization problems, which called Bloch Quantum-behaved Pigeon-Inspired Optimization, BQPIO for short reference. Comparative experimental results verified the feasibility and effectiveness of our proposed approach. Our future work will apply our proposed BQPIO model to solve the complicated issues of unmanned aerial vehichles [15], and which is a challenging issue for bio-inspired computation.

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TABLE I. The Test Results

		PSO		QPSO		PIO		QPIO	
		N_p=100	N_p=20	N_p=100	N_p=20	N_p=100	N_p=20	N_p=100	N_p=20
f1	Mean value	-186.4318	-186.3972	-186.5501	-186.4439	-186.7309	-165.018	-186.7309	-186.7309
	Worst value	-186.337	-186.094	-186.409	-186.3785	-186.7309	-52.0504	-186.7309	-186.7309
f2	Mean value	7.49E-04	1.12E-02	0	3.35E-03	0.36804	1.23383	0	0
	Worst value	1.30E-03	4.73E-02	1.95E-04	1.22E-02	3.4814	5.3395	0	0
f3	Mean value	0	1.80E-03	0	0	0	0.1145	0	0
	Worst value	6.14E-04	5.00E-03	1.08E-04	7.17E-04	0	0.995	0	0
f4	Mean value	0	0	0	0	0	7.93E-02	0	0
	Worst value	1.48E-04	3.69E-04	0	4.76E-03	0	0.1047	0	0