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COMMENTARY

Comments on "Particle swarm optimization with fractional-order velocity"

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Abstract In this paper, some comments on the paper Solteiro Pires et al. (Nonlinear Dyn. 67:893–901, 2010) are presented. We demonstrate that the authors of the above paper have deduced the incorrect formula about the velocity updating strategy of the fractional-order particle swarm optimization algorithm. This paper deduces the modified updating formula, and verified experiments are also conducted.

Keywords Fractional-order calculus · Particle swarm optimization

In Ref. [1], the authors have proposed an improved version of particle swarm optimization (PSO) using fractional-order calculus concepts, in which fractional calculus is used to control its convergence. During the past several years, the fractional-order PSO algorithm has attracted the attention of several researchers [2–4], and many new researches on PSO model improvement have also conducted [5–7].

One error occurred in their designed fractional-order PSO model, which is described in the following section.

As for the original PSO algorithm, the particle movement is characterized by two vectors, namely the current position x and the velocity v. At time t, each particle updates its velocity by the following equation:

$$v_{t+1} - v_t = \phi_1(b - x) + \phi_2(g - x), \quad (1)$$

where b denotes the best position found by the particle so far, and g denotes the global best position achieved by the whole swarm so far. ϕ_1 and ϕ_2 are the randomly uniformly generated terms. For simplicity, symbols and notation are employed with the same meanings as those in Ref. [1]. From the classical integer-order area, the fractional-order PSO algorithm extends the velocity derivative to the fractional-order area, yielding

$$D^\alpha[v_{t+1}] = \phi_1(b - x) + \phi_2(g - x). \quad (2)$$

Thus, Pires et al. [1] derive the new velocity updating strategy as shown below:

$$\begin{aligned} &v_{t+1} - \alpha v_t - \frac{1}{2}\alpha v_{t-1} - \frac{1}{6}\alpha(1-\alpha)v_{t-2} \\ &-\frac{1}{24}\alpha(1-\alpha)(2-\alpha)v_{t-3} \\ &= \phi_1(b - x) + \phi_2(g - x). \end{aligned} \quad (3)$$

According to Ref. [1], (1) is the special case of (3) when $\alpha = 1$. However, it can be observed that (1) cannot be deduced from (3). Therefore, the key formula of the fractional-order PSO algorithm in Ref. [1] is wrong.

To correct the formula, we substitute the following discrete time implementation expression of fractional differential into (2) again.

$$D^\alpha[x(t)] = \frac{1}{T^\alpha}\sum_{k=0}^{r}\frac{(-1)^k\Gamma(\alpha+1)x(t-kT)}{\Gamma(k+1)\Gamma(\alpha-k+1)}, \quad (4)$$

where T is the sampling period and r is the truncation order. $r = 4$ is used in this paper, which is in agreement with Ref. [1].

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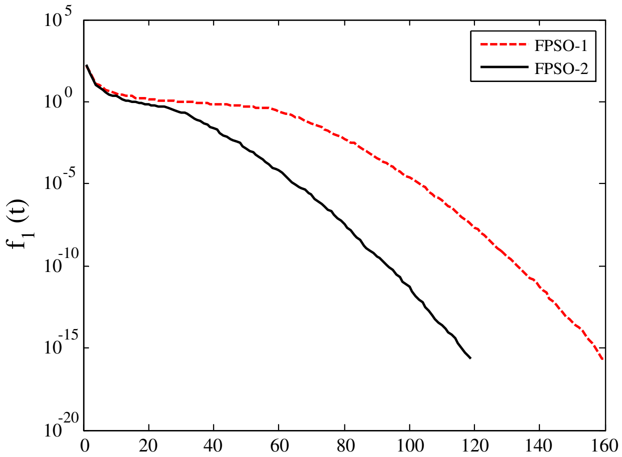

Fig. 1 Evolution of the Bohachevsky 1 function using the FPSOs

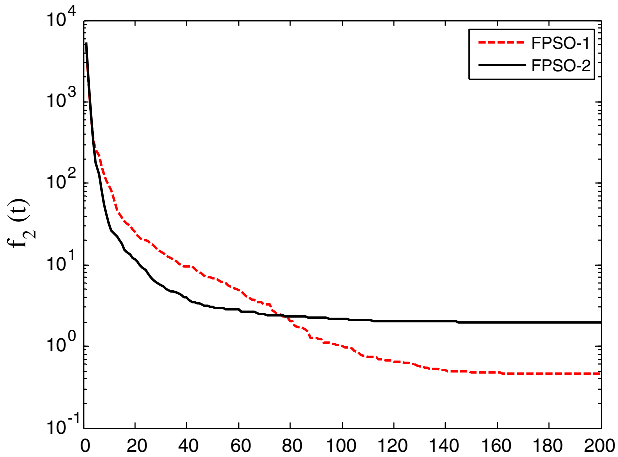

Fig. 2 Evolution of the Colville function using the FPSOs

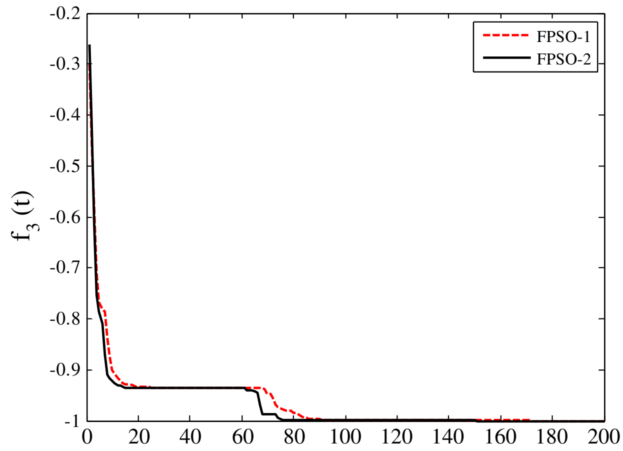

Fig. 3 Evolution of the Drop wave function using the FPSOs

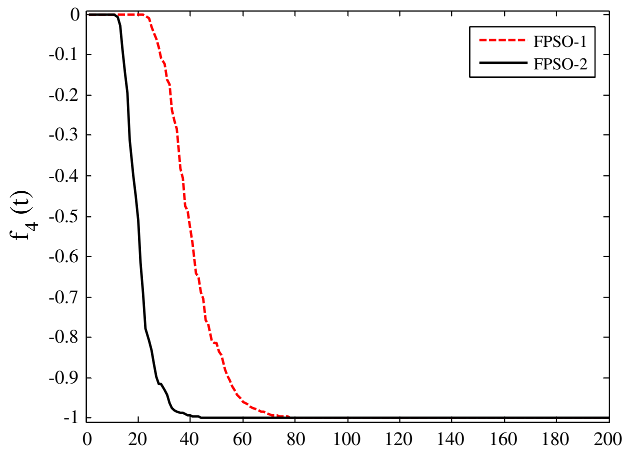

Fig. 4 Evolution of the Easom function using the FPSOs

Hence, the fractional-order behavior of PSO can be written as

$$\begin{aligned} &v_{t+1}-\alpha v_t-\frac{1}{2}\alpha(1-\alpha)v_{t-1}-\frac{1}{6}\alpha(1-\alpha)(2-\alpha)v_{t-2}\\ &-\frac{1}{24}\alpha(1-\alpha)(2-\alpha)(3-\alpha)v_{t-3}\\ &=\phi_1(b-x)+\phi_2(g-x) \end{aligned} \tag{5}$$

That is

$$\begin{aligned} &v_{t+1}=\alpha v_t+\frac{1}{2}\alpha(1-\alpha)v_{t-1}+\frac{1}{6}\alpha(1-\alpha)(2-\alpha)v_{t-2}\\ &+\frac{1}{24}\alpha(1-\alpha)(2-\alpha)(3-\alpha)v_{t-3}\\ &+\phi_1(b-x)+\phi_2(g-x) \end{aligned} \tag{6}$$

In the following sections, we revalidate the performance of the fractional-order PSO algorithm using (6), and the results are compared with those obtained by Pires et al. in Ref. [1]. To indicate the difference, we denote the PSO algorithm described in this comment as FPSO-2, while the algorithm presented in Ref. [1] is denoted by FPSO-1. The test functions adopted herein are the five well-known functions namely Bohachevsky 1, Colville, Drop wave, Easom, and Rastrigin, which are the same expressions as presented in Ref. [1]. Parameters of the FPSO algorithms are also in agreement with Ref. [1] as well, which are set as follows: the population size is 10, the maximum number of iteration is 200, and ϕ_1 and ϕ_2 are randomly uniformly generated in [0, 1]. Moreover, the value of α reduces according to $\alpha(t) = 0.9-0.6t/200$, $t = 0, 1, \ldots, 200$. For the purpose of reducing statistical errors, each algorithm is tested 201 times independently for every function and the median results are used in the comparison. Figures 1, 2, 3, 4 and 5 demonstrate the iteration evolutionary progresses. The correct results for the PSO

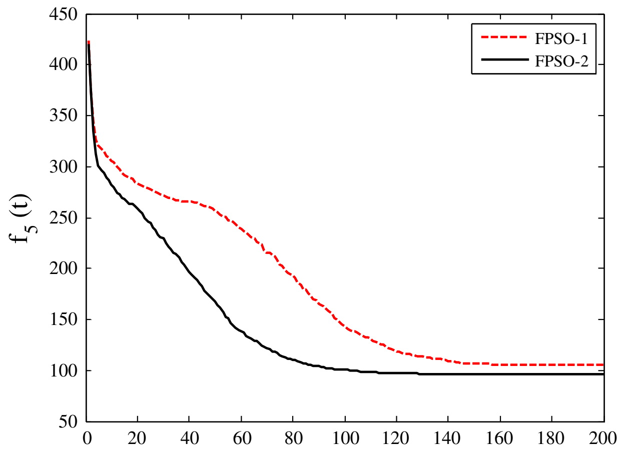

Fig. 5 Evolution of the Rastrigin function using the FPSOs

with fractional-order velocity are indicated by black solid lines.

Moreover, the global minimum value of the Droop wave function is $f^*(x) = -1.0$, rather than $f^*(x) = 0.0$ given by Ref. [1].

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