Orthogonal Multiobjective Chemical Reaction Optimization Approach for the Brushless DC Motor Design

Haibin Duan and Lu Gan

State Key Laboratory of Virtual Reality Technology and Systems, Beihang University, Beijing 100191, China
Science and Technology on Aircraft Control Laboratory, School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China

The optimal design of a brushless direct-current motor (BLDCM) is a prevalent and practical issue in the field of magnetics. A major problem is to design a BLDCM so that it operates optimally in the sense of producing maximum efficiency with minimal material cost. Using the sizing model, a novel orthogonal multiobjective chemical reaction optimization (OMOCRO) algorithm is proposed to solve this problem. Chemical reaction optimization is a newly proposed heuristic algorithm, inspired by the interactions between molecules during chemical reactions. In our proposed OMOCRO, we employ a Pareto ranking scheme to deal with multiobjective optimization problems, and orthogonal experimental design is used in the initialization stage. Comparative experiments with nondominated sorting genetic algorithm and multiobjective particle swarm approach demonstrate that our proposed method is more competitive in handling complex optimization problems.

***Index Terms*— Brushless direct-current motor (BLDCM), chemical reaction optimization (CRO), multiobjective optimization, orthogonal experimental design (OED).**

I. Introduction

BRUSHLESS direct-current motors (BLDCMs) with the advantages like simple structure, light weight, and high efficiency are now widely used in modern industry. Thus, a great deal of research effort has been devoted to the design of a BLDCM in recent decades [1]–[5]. A new method to solve this problem is using evolutionary strategy and swarm intelligence optimization algorithms.

Generally, the optimal design of a BLDCM is a multiobjective optimization problem with several variables and constraints. There are several methods dealing with this problem in [3]–[5] by using multiobjective particle swarm approach (MOPSO) [3], the modified nondominated sorting genetic algorithm (NSGA-II), and sequential quadratic programming (SQP) [4]. In [5], a biologically inspired approach namely bat algorithm (BA) is adopted to optimize the design of a BLDCM.

Chemical reaction optimization (CRO) is a new metaheuristic algorithm, imitating the interactions of molecules in chemical reactions to reach the global optimum. Although CRO was recently proposed by Lam and Li [6], it has been successfully employed to solve many mono-objective optimization problems. Multiobjective optimization is more important from the perspective of real-world problem solving, such as efficiency and total mass, in a BLDCM design problem.

In this paper, a variant of CRO-named multiobjective chemical reaction optimization (MOCRO) is proposed to optimally design a BLDCM because of two aspects. On the one hand, the reactants–products property of CRO is analogous to the parents–offspring property of NSGA-II, which can be used to solve multiobjective optimization problems. On the other hand, it is demonstrated that CRO yields state-of-the-art results in solving many practical problems and CRO is more tractable.

Therefore, MOCRO is applied to optimize the design of a BLDCM for the sake of a better solution. In addition, we employ an Orthogonal Experimental Design (OED) with quantization technique [7] to generate a cluster of scattered initial molecules, so that MOCRO can scan uniformly the feasible solution space to find a good Pareto frontier.

The organization of this paper is as follows. Section II formulates the brushless dc motor design problem. Section III explains the principles of basic CRO. In Section IV, we describe the design of orthogonal multiobjective CRO (OMOCRO). Section V provides experimental results and theoretical analyses. Concluding remarks are finally given in Section VI.

II. Problem Formulation

The analytical model developed in [8] is used in this paper. As it is composed of numerous nonlinear equations, the design of this model becomes a benchmark with five design parameters (i.e., optimization parameters), five fixed variables, and six constraints as described in [9]. Variables are shown in Table I except the fixed ones which we do not elaborate here. Efficiency, total mass, and the constraint variables are all determined by the design parameters.

In our related paper, we use brain storm optimization (BSO) to maximize the motor's efficiency with a constraint total mass, which is a mono-objective optimization problem [10]. Now, this paper deals with the multiobjective optimization problem, in which the objective function is a combination of loss $(1-\eta)$ and total mass to be minimized simultaneously when the constraints are maintained.

Manuscript received October 26, 2013; revised January 29, 2014 and April 13, 2014; accepted May 15, 2014. Date of publication May 20, 2014; date of current version January 26, 2015. Corresponding author: H. Duan (e-mail: hbduan@buaa.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TMAG.2014.2325797

TABLE I
VARIABLES IN THE MOTOR MODEL

Variable	Symbol	Physical Meaning	Quantity
Objectives	η (%)	Efficiency	—
	M_{tot}(kg)	Total mass	—
Design Parameters	D_S(mm)	Bore diameter	[150, 330]
	B_e(T)	Maximum magnetic induction in the air gap	[0.5, 0.76]
	δ (A/mm^2)	Density current on the windings	[2.0, 5.0]
	B_d(T)	Average magnetic induction in the teeth	[0.9, 1.8]
	B_{cs}(T)	Average magnetic induction in the stator back iron	[0.6, 1.6]
Constraints	D_{ext}(mm)	Outer diameter	D_{ext}<340
	D_{int}(mm)	Inner diameter	D_{int} >76
	I_{max}(A)	Maximum current in the phases	I_{max} >125
	discr(Ds, δ , B_d, B_e)	Determinant used for the calculation of the slot height	*discr* >0
	T_a(℃)	Motor Temperature	T_a<120

This problem can be mathematically described as follows:

$$\begin{aligned}&\text{Minimize } F = [(1-\eta), M_{\text{tot}}]\\ &\text{with } 150\text{ mm} < D_s < 330\text{ mm}, 0.5T < B_e < 0.76T\\ &\quad 2.0\ A/\text{mm}^2 < \delta < 5.0\ A/\text{mm}^2, 0.9T < B_d < 1.8T\\ &\quad 0.6T < B_{cs} < 1.6T\\ &\text{s.t. } D_{\text{ext}} < 340\text{ mm}, D_{\text{int}} > 76\text{ mm}, I_{\max} > 125A\\ &\quad \text{discr}(D_s, \delta, B_d, B_e) > 0, T_a < 120\ ^\circ C. \end{aligned} \quad (1)$$

III. CHEMICAL REACTION OPTIMIZATION

CRO is a population-based metaheuristic algorithm, inspired by the process of a chemical reaction where molecules interact with each other through collisions and change from high- to low-energy states. In this section, we will first introduce the property of molecules which are basic operating agents of CRO. Then a law CRO strictly abides by is presented. This section ends with the descriptions of elementary reactions which are different ways of manipulating the energies of the involved molecule(s).

A. Molecules

CRO involves a pool of molecules to explore the solution space, and each molecule contains a set of attributes as given in Table II. The last four attributes are designed to avoid CRO getting stuck in a local optimal, and this process is described in [6] and [11]. According to the problems, we can optionally introduce other attributes into molecules.

TABLE II
ATTRIBUTES OF A MOLECULE

Symbol	Chemical meaning	Mathematical meaning
ω	Molecular structure	Solution of a problem
PE_ω	Potential energy	Objective function value
KE_ω	Kinetic energy (non-negative)	Measure of tolerance of accepting a worse solution
NumHit	Number of hits	Current total number of hits
MinStruct	Minimum structure	Current optimal solution
MinPE	Minimum PE	Current optimal function value
MinHit	Minimum hit number	Number of hits when the current optimal solution is found

TABLE III
DESIGN OF ELEMENTARY REACTIONS

Reaction	Operator
On-wall Ineffective Collision	$\omega \rightarrow \omega'$ [a]
Inter-molecular Ineffective Collision	$\omega_1 + \omega_2 \rightarrow \omega'_1 + \omega'_2$ [b]
Decomposition	$\omega \rightarrow \omega'_1 + \omega'_2$ [c]
Synthesis	$\omega_1 + \omega_2 \rightarrow \omega'$ [d]

ω, ω_1, ω_2 are reactants (input molecules) and ω', ω'_1, ω'_2 are products in a reactions (output molecules). Each type of reaction must abide by the following terms.

[a] $E_{excess} = PE_\omega + KE_\omega - PE_{\omega'} > 0$

[b] $E_{excess} = PE_{\omega_1} + KE_{\omega_1} + PE_{\omega_2} + KE_{\omega_2} - PE_{\omega'_1} - PE_{\omega'_2} > 0$

[c] $E_{excess} + E_{buffer} = PE_\omega + KE_\omega - PE_{\omega'_1} - PE_{\omega'_2} + E_{buffer} > 0$

[d] $E_{excess} = PE_{\omega_1} + KE_{\omega_1} + PE_{\omega_2} + KE_{\omega_2} - PE_{\omega'} > 0$

B. Energy Conservation Law

CRO mimics a chemical reaction in a closed container which could be regarded as an isolated system. Real system is governed by the energy conservation law, so a reaction occurs in CRO when the following equations are satisfied:

$$\begin{aligned}E'_{\text{total}} &= E'_{\text{buffer}} + PE_{\omega'1} + KE_{\omega'1} + PE_{\omega'2}\\ &\quad + KE_{\omega'2} + \cdots + PE_{\omega'n} + KE_{\omega'n}\\ &= E_{\text{buffer}} + PE_{\omega1} + KE_{\omega1} + PE_{\omega2} + KE_{\omega2}\\ &\quad + \cdots + PE_{\omega m} + KE_{\omega m} = E_{\text{total}}\end{aligned} \quad (2)$$

where E is the energy, and n and m are the number of molecules after and before a reaction, respectively.

C. Elementary Reactions

There are four kinds of elementary reactions defined in CRO, depicted briefly in Table III.

How products are generated from reactants is designed according to the problem. The reactions mechanism adopted for the problems in this paper is described in what follows. We add a Gaussian perturbation to the former molecule to get a new one in an on-wall ineffective collision and decomposition. For example, $\omega' = \omega + \delta$, where δ is the permutation vector of random variables with a Gaussian probability density function having designed mean and variance. Operators based on probability are applied to the intermolecular ineffective collision

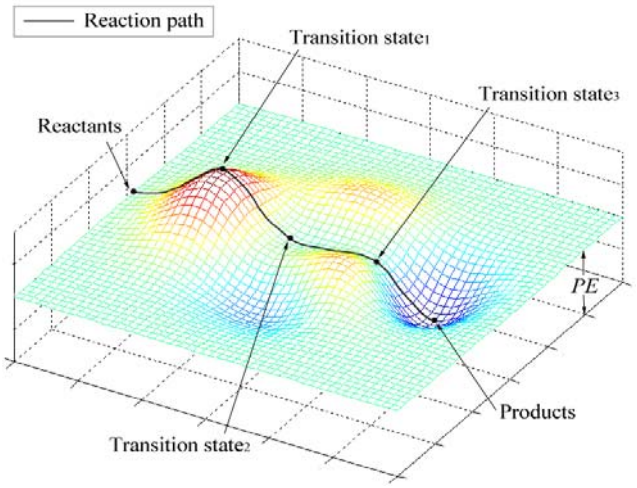

Fig. 1. Illustration of a chemical reaction on the potential energy surface.

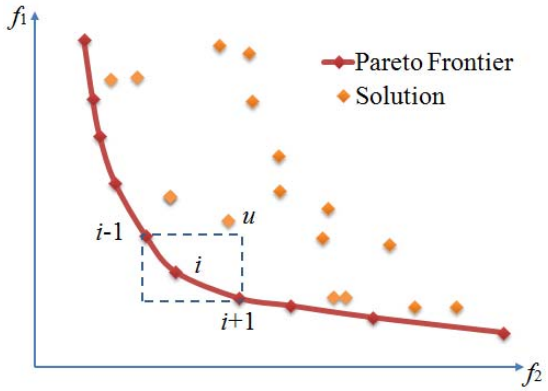

Fig. 2. Sketch of a Pareto frontier and the crowding distance.

and synthesis. In an intermolecular ineffective collision, we randomly exchange some elements of the former molecules to get products. In a synthesis, if a random number is greater than a threshold, $\omega' = \omega_1$, otherwise, $\omega' = \omega_2$.

Generally, the two ineffective collisions implement local search while the other reactions give the effect of diversification. When CRO determines that a molecule is stuck in a local minimum, more intense reactions (i.e., decomposition and synthesis) happen to help it jump out. A chemical reaction occurs among molecules when they are unstable with excessive energy, and stops when they possess low potential energy. This process can be seen in Fig. 1. Other important parameters in CRO are described in [6] and [11].

IV. Orthogonal Multiobjective CRO

A. Pareto Ranking Scheme

Pareto ranking scheme is a strategy involved in many algorithms to handle multiobjective optimization problems, such as Pareto simulated annealing (PSA), MOPSO, and NSGA-II. In Pareto criterion, a solution vector $\mathbf{y}$ dominates another vector $\mathbf{x}$ when the following conditions are satisfied:

$$\begin{cases} f_i(\mathbf{y}) \le f_i(\mathbf{x}) & \text{for all } i = 1, 2, 3, ..., n \\ f_{\bar{i}}(\mathbf{y}) < f_{\bar{i}}(\mathbf{x}) & \text{for at least one } \bar{i} \in \{1, 2, 3, ..., n\} \end{cases} \quad (3)$$

For $i = 1$: *iniPopSize*
  Generate ω_i in Pop_1
End for
$t = 1$
$RepSize = iniPopSize$
$Rep_1 = \varnothing$
While ($t < Repeats$)
  $NumHit = 1$
  While ($NumHit < FE$)
    Select one or two molecule(s) in Pop_t
    Select a kind of elementary reactions
    Complement a chemical reaction
    $NumHit = NumHit + 1$
  End while
  $Rep_t = Rep_t \cup Pop_t$
  If ($Size(Rep_t) > RepSize$)
    Crowding distance assignment (Rep_t)
    Sort (Rep_t, descend)
    Select the previous *RepSize* molecules in Rep_t as Rep_t
    $Pop_{t+1} = Rep_t$
  End if
  $t = t + 1$
End while

Fig. 3. Pseudocode of our proposed MOCRO.

where n is the number of objective functions. A Pareto optimal solution in a problem is such a set of solutions, each of which is in the first nondomination front (see Fig. 2). In this figure, solutions $i-1$ and $i+1$ are not dominated by each other because $f_2(i-1) < f_2(i+1)$, $f_1(i+1) < f_1(i-1)$. Solution u is not in the Pareto frontier since u is dominated by solution $i-1$ and $i+1$.

Deb *et al*. proposed a technique that combines fast nondominated sorting approach with crowded-comparison operator in NSGA-II [12]. Fast nondominated sorting approach is based on two entities which are the number of solutions that dominate the solution i (n_i), and a set of solutions that the solution i dominates (S_i). By repeatedly visiting each solution in S_i, reducing n_i, and comparing n_i with zero, all fronts are finally identified with a computational complexity of $O(MN^2)$ instead of $O(MN^3)$ in a naive sorting procedure, where M is the number of objectives and N is the population size.

In this technique, crowded-comparison operator is executed to preserve the diversity of solutions and obtain a uniform Pareto frontier. Crowding distance of solution i is defined in

$$F(i)_{\text{distance}} = F(i)_{\text{distance}} + \frac{F(i+1).n - F(i-1).n}{f_n^{\max} - f_n^{\min}} \quad (4)$$

where $F(i).n$ is the value of the nth objective function of solution i in front F, and $f_n^{\max}$ and $f_n^{\min}$ are the maximum and minimum of the nth objective function, respectively. Selecting solutions with larger crowding distance can maintain the diversity of the solutions.

Therefore, fast nondominated sorting procedure gives the rank of all solutions, and crowded-comparison operator is used to evaluate solutions in the same rank. Among the solutions in the same rank, solutions with larger crowding distances are better than solutions with smaller crowding distances in terms

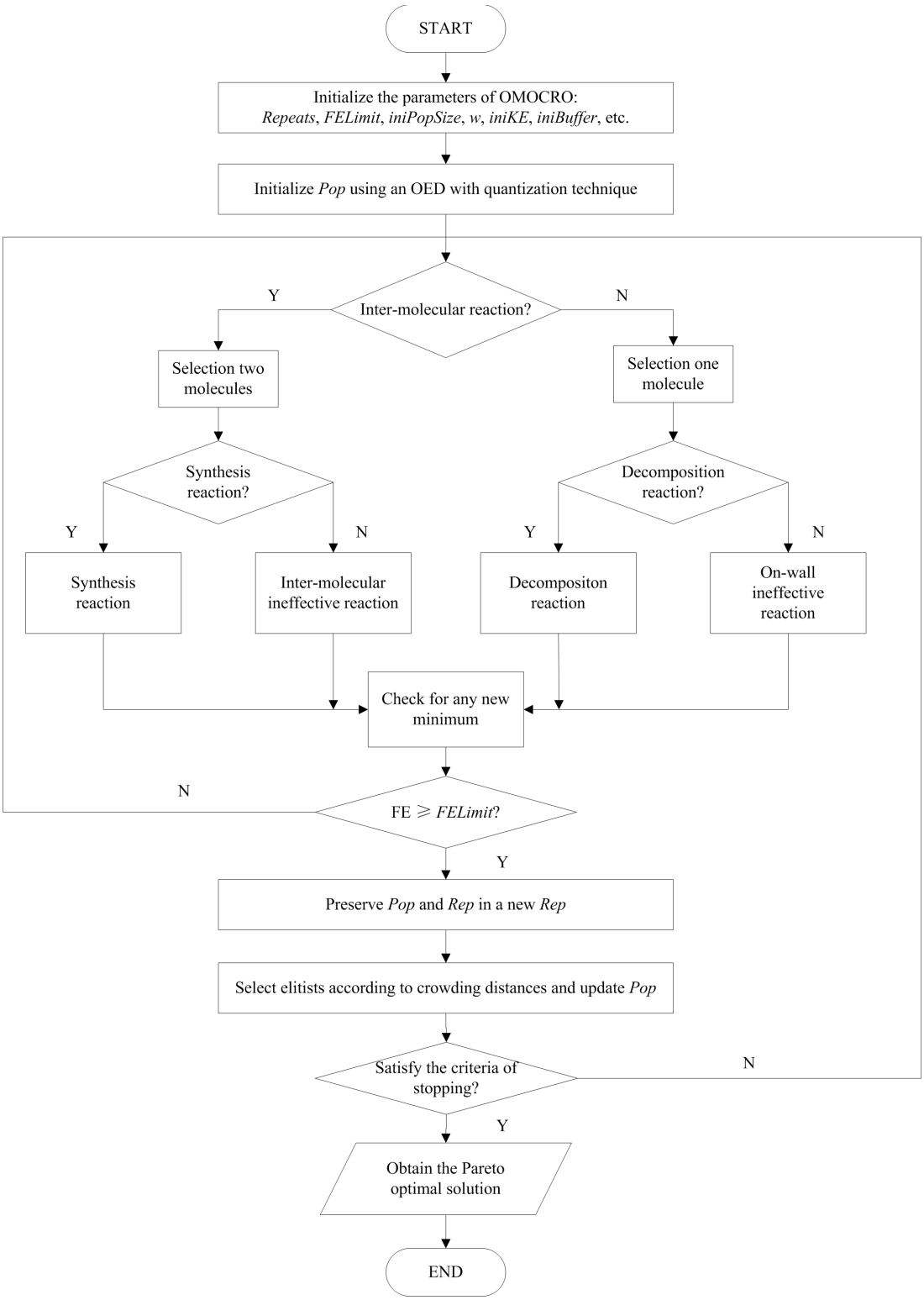

Fig. 4. Schematic diagram of OMOCRO.

of diversity. This technique has been proved very effective and efficient in NSGA-II and some other algorithms to execute an elitist selection, so we introduce them into our MOCRO.

B. Elitist Strategy in MOCRO

In MOCRO, we initialize a pool of molecules (Pop) with attributes representing the values of five design parameters (D_S, B_e, δ, B_d, B_{cs}), and two objective functions ($1-\eta$, M_{tot}). We assign the potential energy (PE) of a molecule ω as its nondomination rank as follows:

$$\text{PE}_\omega = w \cdot \text{Rank}_\omega \tag{5}$$

where w is a weight coefficient. When the number of hits (NumHit) reaches the parameter function evaluations (FE),

TABLE IV
ELEMENTARY PARAMETERS OF THE THREE ALGORITHMS

Algorithm	Symbol	Algorithmic Meaning	Quantity
Orthogonal MOCRO	*iniPopSize*	Number of molecules in *Pop*	50
	w	weight coefficient	100
	FELimit	Number of reactions in a repeat	50
	Repeats	Maximum times of repeats	200
	iniKE	Initial kinetic energy in each molecule	100
	iniBuffer	Initial energy in the center buffer	1000
	decThres	Threshold defined for decomposition	30
	synThres	Threshold defined for synthesis	10
	lossRate	Maximum percentage of *KE* lost at a time	0.5
	collRate	Occurrence rate of an inter-molecular reaction	0.3
NSGA-II	*Pop*	Size of population	50
	Gen	Maximum number of iterations	200
	poolSize	Size of a mating pool after tournament selection	25
	tourSize	Size of the tournament	2
	mu	Distribution index for crossover	20
	mum	Distribution index for mutation	20
MOPSO	*nPop*	Size of population	50
	nRep	Size of repository	50
	MaxIt	Maximum number of iterations	200
	ω	Inertia weight	0.7298
	c_1	Personal learning coefficient	1.4962
	c_2	Global learning coefficient	1.4962

we can obtain the Pop after chemical reactions. Then we conserve molecules on the current Pareto frontier by holding them in Rep whose size is equal to that of Pop, and start reactions again with Pop. When the size of Rep exceeds a fixed value, a fast crowding distance estimation procedure is conducted on Rep to select elitists among them. The process is shown in Fig. 3.

After a series of reactions finished, Pop are assigned as superior Rep and involved in a new round. After several loops, the Pareto optimal set is obtained.

C. Orthogonal Experimental Design

Although there is a crowded-comparison operator in MOCRO, the final Pareto optimal set can be distributed nonuniformly sometimes, because the Pop in MOCRO are generated in a stochastic way and usually search nearby. If the positions of initial Pop are not reasonable, the solutions finally obtained could be not reasonable either. To address this problem, we incorporate OED in the initialization stage.

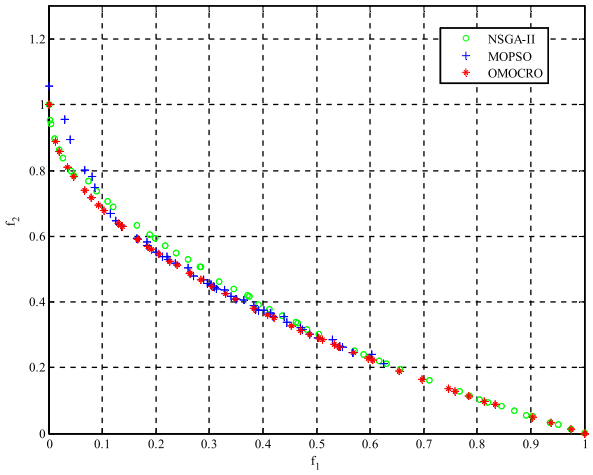

Fig. 5. Test solutions with NSGA-II, MOPSO, and OMOCRO on ZDT1.

OED is an experimental design method merged into evolution algorithms to sample representative combinations of factors in a statistical manner. OED performs well in cooperating with GA [7], SA [13], and PSO [14]. In this paper, we apply an OED with quantization technique [7] in our MOCRO to generate initial molecules. The procedure is described below.

Step 1 (Construction of Orthogonal Array): There are five factors (i.e., five design parameters in the motor model) and three levels per factor we defined in this problem. So we construct an orthogonal array $L_{3^3}(3^5) = [a_{i,j}]_{3^3\times 5}$ according to [7, Algorithms 1–2].

Step 2: Divide the feasible solution space [**low**, **up**] into several subspaces by using

$$\begin{cases} \mathbf{low}(i) = \mathbf{low} + (i-1)\left(\frac{\mathrm{up}_s - \mathrm{low}_s}{S}\right)\mathbf{1}_s \\ \mathbf{up}(i) = \mathbf{up} - (S-1)\left(\frac{\mathrm{up}_s - \mathrm{low}_s}{S}\right)\mathbf{1}_s \end{cases} \quad i = 1, 2, \ldots, S \tag{6}$$

where S is the quantity of subspaces, $\mathbf{1}_s$ is a 5-D vector whose sth element is one, and the others are zero. s is defined as

$$\mathrm{up}_s - \mathrm{low}_s = \max_{1\le i\le 5}\left\{\mathrm{up}_n - \mathrm{low}_n\right\}. \tag{7}$$

Step 3: For each subspace [**low**(i), **up**(i)] $i = 1, 2, \ldots, S$, quantize them based on

$$\alpha_{n,m} = \begin{cases} \mathrm{low}_n & m = 1 \\ \mathrm{low}_n + (m-1) & 2 \le m \le Q-1, \ n = 1, 2, \ldots, N \\ \mathrm{up}_n & m = Q \end{cases} \tag{8}$$

where Q and N are the level and factor value which are equal to three and five, respectively, in this paper.

Step 4 (Generation of Initial Pop): Molecules in the first subspace are produced as follows:

$$\begin{cases} (\alpha_{1,a_{1,1}}, \alpha_{2,a_{1,2}}, \ldots, \alpha_{5,a_{1,5}}) \\ (\alpha_{1,a_{2,1}}, \alpha_{2,a_{2,2}}, \ldots, \alpha_{5,a_{2,5}}) \\ \cdots \\ (\alpha_{1,a_{27,1}}, \alpha_{2,a_{27,2}}, \ldots, \alpha_{5,a_{27,5}}). \end{cases} \tag{9}$$

TABLE V
PERFORMANCE COMPARISON OF THE THREE ALGORITHMS FOR ZDT1

Algorithm	D-metric	Δ-metric	∇-metric
NSGA-II	-	**3.9263e-1**	1.8752
MOPSO	8.4635e-2	7.4223e-1	1.4695
OMOCRO	**7.6281e-2**	4.0410e-1	**1.9438**

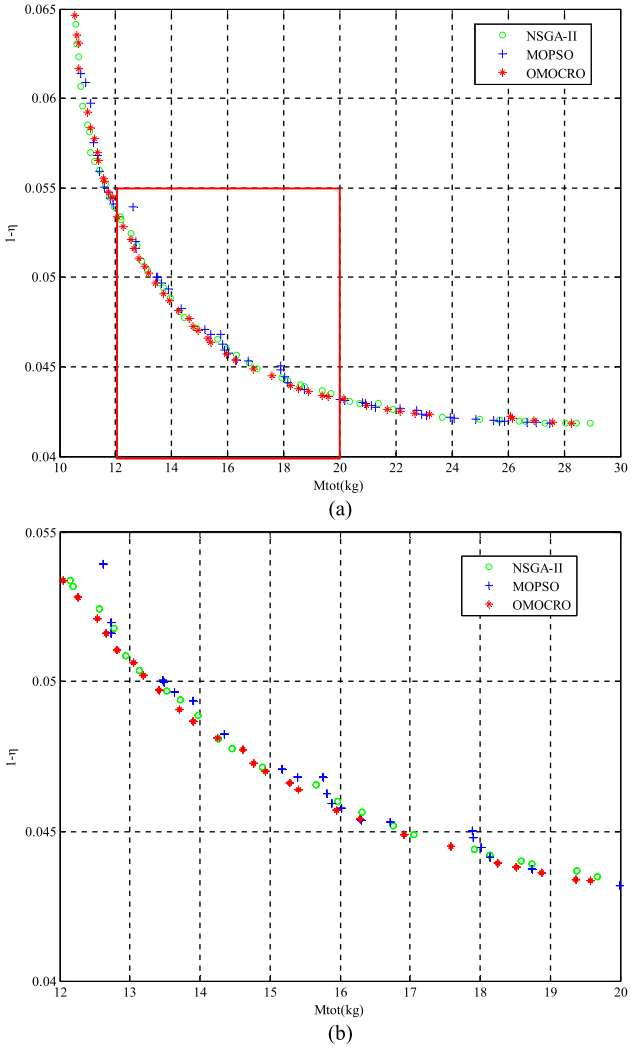

Fig. 6. Comparative results of OMOCRO, NSGA-II, and MOPSO. (a) Pareto frontier found by OMOCRO, NSGA-II, and MOPSO. (b) Details in the red rectangle of (a).

Each subspace generate 27 molecules, so we will get $27 \times S$ molecules altogether. Finally, we put them into a nondominated sorting procedure to select molecules with minimal nondominating rank as the initial Pop and start MOCRO. To summarize the algorithm, we give the schematic diagram of our proposed OMOCRO as shown in Fig. 4.

V. SIMULATION

Before optimizing the design of a BLDCM, we firstly test our OMOCRO by using the ZDT1 problem, which is a benchmark optimization problem proposed in [12]

TABLE VI
SOLUTIONS OBTAINED BY THE THREE ALGORITHMS

	η (%) (maximum)			M_{tot}	
NSGA-II	**95.8161**			28.9275	
	D_S	B_e	δ	B_d	B_{cs}
	282.0733	0.7087	2.0000	1.2159	0.6000
	η (%)			M_{tot} (minimum)	
	93.5366			10.5623	
	D_S	B_e	δ	B_d	B_{cs}
	187.8222	0.6491	3.6762	1.8000	1.6000
	η (%) (maximum)			M_{tot}	
	95.7741			26.0793	
	D_S	B_e	δ	B_d	B_{cs}
MOPSO	278.7818	0.6041	2.0000	1.1147	0.6000
	η (%)			M_{tot} (minimum)	
	93.9309			10.7624	
	D_S	B_e	δ	B_d	B_{cs}
	182.6379	0.6528	3.3227	1.7917	1.5423
	η (%) (maximum)			M_{tot}	
	95.8134			28.0367	
	D_S	B_e	δ	B_d	B_{cs}
OMOCRO	275.9669	0.7076	2.0000	1.2134	0.6000
	η (%)			M_{tot} (minimum)	
	93.4891			**10.5551**	
	D_S	B_e	δ	B_d	B_{cs}
	188.5973	0.6476	3.7172	1.7998	1.5966

$$\text{ZDT1}: \begin{cases} \text{Minimize } f_1(\mathbf{x}) = x_1 \\ \text{Minimize } f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - \sqrt{\frac{x_1}{g(\mathbf{x})}}\right] \\ g(\mathbf{x}) = 1 + 9\frac{\sum_{i=2}^{n} x_i}{n-1} \\ 0 \le x_i \le 1,\ i = 1, 2, .., n \end{cases} \tag{10}$$

where n is the number of decision variables.

In this function test and the following real-world problem experiment, OMOCRO is compared with NSGA-II and MOPSO. The important parameters in the three algorithms are given in Table IV. Because only one or two molecules participate in a reaction at the same time, we define FELimit as the size of population in NSGA-II and MOPSO in each repeat to simulate a generation. Test results are shown in Fig. 5.

Owing to convergence properties of CRO and the OED with quantization technique, OMOCRO exhibits best performance compared with NSGA-II and MOPSO. Intuitively, OMOCRO finds better convergence and spread of solutions than NSGA-II and MOPSO shown in Fig. 5. To take a deep sight into the test results, we present a quantitative analysis of solutions by calculating the average of three performance metrics for 50 trials of each algorithm. The statistic results are given in Table V, in which metrics are computed according to [15]. The Pareto frontier for NSGA-II is taken as the reference set for the calculation of the D-metric, that is, the convergence metric.

Results in Table V show that in terms of the Δ-metric (diversity metric), the average of OMOCRO and NSGA-II are very close owing to crowding distance assignment. In terms of the D-metric and ∇-metric (extent metric), OMOCRO outperforms MOPSO and NSGA-II. In conclusion, the performance

of OMOCRO is better or at least similar to NSGA-II, and clearly superior to MOPSO.

Then, the BLDCM design problem is optimized by the three algorithms. In this problem, each algorithm runs 600 iterations and other parameters are the same as those given in Table IV. Molecules which do not satisfy the constraints in Table I represent unfeasible solutions in this problem, so these molecules should be directly abandoned. Specifically, the efficiency values of these molecules are set to 0 and the total mass values are set to 30. When the evolution process finished (the maximum iteration number is achieved), the design parameters are examined by their limits. If they exceed the limits, random values within the allowable range will substitute them.

Pareto optimal solutions are shown in Fig. 6. It is clear that a large proportion of solutions in the Pareto frontier obtained by OMOCRO dominate solutions obtained by NSGA-II and MOPSO [see Fig. 6(b)]. In harmony with the analysis shown in Table V, the Pareto frontier for OMOCRO shows high validity and uniformity.

Finally, the best feasible solutions obtained by these algorithms are presented in Table VI. All results fulfill the constraints. As can be seen, although the maximum efficiency result from MOCRO is little less than that from NSGA-II, but the corresponding total mass is much less than the latter. And the minimum total mass solution is obtained by OMOCRO.

VI. Conclusion

In this paper, a novel multiobjective optimization method OMOCRO needed in optimal design of the BLDCM has been proposed. CRO is a recently developed algorithm and its convergence properties have been theoretically analyzed by modeling it as a Markov chain in [16].

Efficiency and total mass of the BLDCM are the two objectives in the optimization problem considering the design constraints of diameters, maximum current, temperature, and so on. OMOCRO has been used to find the optimal design of the motor. Comparative simulation results demonstrate that the proposed algorithm is a suitable method for solving the motor design problem, and is competitive with current evolutionary techniques. Furthermore, the development of novel optimization methods more fitted to motor models is an important issue in magnetics, and we will try to apply the presented algorithm to the motor driving systems and path planners [17] of unmanned aerial vehicles. We will also try to apply a novel pigeon-inspired optimization for solving the dc motor design problems [18].

Acknowledgments

This work was supported in part by the National Key Basic Research Program of China (973 Project) under Grant 2013CB035503 and Grant 2014CB046401, in part by the Natural Science Foundation of China under Grant 61333004, Grant 61273054, and Grant 61175109, in part by the Top-Notch Young Talents Program of China, in part by the National Magnetic Confinement Fusion Research Program of China under Grant 2012GB102006, and in part by the Graduate Innovation Foundation of Beihang University under Grant YCSJ-01-2014-01.

References

[1] M. Markovic and Y. Perriard, "Optimization design of a segmented Halbach permanent-magnet motor using an analytical model," *IEEE Trans. Magn.*, vol. 45, no. 7, pp. 2955–2960, Jul. 2009.

[2] S.-M. Jang, H.-W. Cho, and S.-K. Choi, "Design and analysis of a high-speed brushless DC motor for centrifugal compressor," *IEEE Trans. Magn.*, vol. 43, no. 6, pp. 2573–2575, Jun. 2007.

[3] L. dos Santos Coelho, L. Z. Barbosa, and L. Lebensztajn, "Multiobjective particle swarm approach for the design of a brushless DC wheel motor," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 2994–2997, Aug. 2010.

[4] F. Moussouni, S. Brisset, and P. Brochet, "Some results on the design of brushless DC wheel motor using SQP and GA," *Int. J. Appl. Electromagn. Mech.*, vol. 26, nos. 3–4, pp. 233–241, Sep. 2007.

[5] T. C. Bora, L. dos Santos Coelho, and L. Lebensztajn, "Bat-inspired optimization approach for the brushless DC wheel motor problem," *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 947–950, Feb. 2012.

[6] A. Y. S. Lam and V. O. K. Li, "Chemical-reaction-inspired metaheuristic for optimization," *IEEE Trans. Evol. Comput.*, vol. 14, no. 3, pp. 381–399, Jun. 2010.

[7] Y.-W. Leung and Y. Wang, "An orthogonal genetic algorithm with quantization for global numerical optimization," *IEEE Trans. Evol. Comput.*, vol. 5, no. 1, pp. 41–53, Feb. 2001.

[8] S. Brisset and P. Brochet, "Analytical model for the optimal design of a brushless DC wheel motor," *COMPEL, Int. J. Comput. Math. Electr. Electron. Eng.*, vol. 24, no. 3, pp. 829–848, 2005.

[9] *A Benchmark for a Mono and Multi Objective Optimization of the Brushless DC Wheel Motor*. L2EP, École Centrale de Lille, Villeneuve-d'Ascq, France [Online]. Available: http://l2ep.univ-lille1.fr/come/benchmark-wheel-motor.htm

[10] H. Duan, S. Li, and Y. Shi, "Predator–prey brain storm optimization for DC brushless motor," *IEEE Trans. Magn.*, vol. 49, no. 10, pp. 5336–5340, Oct. 2013.

[11] A. Y. S. Lam and V. O. K. Li, "Chemical reaction optimization: A tutorial," *Memetic Comput.*, vol. 4, no. 1, pp. 3–17, Mar. 2012.

[12] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.

[13] S.-Y. Ho, S.-J. Ho, Y.-K. Lin, and W. C.-C. Chu, "An orthogonal simulated annealing algorithm for large floorplanning problems," *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 12, no. 8, pp. 874–876, Aug. 2004.

[14] S.-Y. Ho, H.-S. Lin, W.-H. Liauh, and S.-J. Ho, "OPSO: Orthogonal particle swarm optimization and its application to task assignment problems," *IEEE Trans. Syst., Man, Cybern. A, Syst., Humans*, vol. 38, no. 2, pp. 288–298, Mar. 2008.

[15] C. Erbas, S. Cerav-Erbas, and A. D. Pimentel, "Multiobjective optimization and evolutionary algorithms for the application mapping problem in multiprocessor system-on-chip design," *IEEE Trans. Evol. Comput.*, vol. 10, no. 3, pp. 358–374, Jun. 2006.

[16] A. Y. S. Lam, V. O. K. Li, and J. Xu, "On the convergence of chemical reaction optimization for combinatorial optimization," *IEEE Trans. Evol. Comput.*, vol. 17, no. 5, pp. 605–620, Oct. 2013.

[17] H. B. Duan and P. Li, *Bio-Inspired Computation in Unmanned Aerial Vehicles*. Berlin, Germany: Springer-Verlag, Jan. 2014.

[18] H. B. Duan and P. X. Qiao, "Pigeon-inspired optimization: A new swarm intelligence optimizer for air robot path planning," *Int. J. Intell. Comput. Cybern.*, vol. 7, no. 1, pp. 24–37, 2014.