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Multi-objective pigeon-inspired optimization for brushless direct current motor parameter design

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Pigeon-inspired optimization (PIO) is a new swarm intelligence optimization algorithm, which is inspired by the behavior of homing pigeons. A variant of pigeon-inspired optimization named multi-objective pigeon-inspired optimization (MPIO) is proposed in this paper. It is also adopted to solve the multi-objective optimization problems in designing the parameters of brushless direct current motors, which has two objective variables, five design variables, and five constraint variables. Furthermore, comparative experimental results with the modified non-dominated sorting genetic algorithm are given to show the feasibility, validity and superiority of our proposed MIPO algorithm.

brushless direct current (BLDC) motor, multi-objective pigeon-inspired optimization (MPIO), electromagnetics, multi-objective optimization

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1 Introduction

Electric vehicles are considered as an attractive option on the pathway towards low-emission vehicles that can enable the transport sector to reduce sectoral greenhouse gas emissions by a significant degree [1]. Hence, general motors still cannot offer high-speed constant-power operation and high efficiency over wide-speed operation range, which also limits the applications to electric vehicles [2]. Brushless Direct Current (BLDC) motors are currently the most popular motor choice for electric vehicles.

Recently, the interest in the BLDC motor design shoots up from both theoretical and experimental points of view. Shiadeh, Ardebili and Moamaei [3] presented two different Axial-Flux Permanent-Magnet (AFPM) BLDC machine topologies with similar pole and slot combination. Dadashnialehi [4] proposed a sensorless Antilock Braking System (ABS) for brushless-motor in-wheel electric vehicles. Norhisam et al. [5] devised a high torque BLDC, which is designed in new arrangement of the stator teeth and operated as three-phase motor.

Meanwhile, Swarm Intelligence (SI) is active in the field of applied electromagnetics with a gorgeous figure. Several bio-inspired swarm intelligent optimization algorithms have been applied to the design of the BLDC motors, such as Bat Algorithm (BA) [6], Predator-Prey Biogeography-Based Optimization (PPBBO) [7], Multi-Objective Particle Swarm Optimization (MOPSO) [8], the modified Non-dominated Sorting Genetic Algorithm (NSGA-II), and Sequential Quadratic Programming (SQP) [9], respectively.

With the development of science and technology, many practical optimization design problems arise, which give rise to the booming of the bio-inspired computation algorithms [10]. After applying several bio-inspired computa-

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tion algorithms to controller design [11,12], Duan and Qiao [13] firstly proposed a novel bio-inspired computing algorithm named Pigeon-Inspired Optimization (PIO) in 2014.

Homing pigeons can easily find their homes by using three homing tools [13]: magnetic field, sun [14] and landmarks. The magnetic field is perceived from magnetite particles carried from the nose to the brain by the trigeminal nerve [15]. Guilford et al. [16] argues that pigeons probably use different navigational tools during different parts of their journeys. They may rely more on compass-like tools at the beginning of journey, while in the middle, they will switch to landmarks and reassess their routes to make corrections. In the novel algorithm, map and compass operator model and landmark operator model are presented based on magnetic field and sun, and landmarks, respectively.

PIO has proven itself as a valuable competitor in some optimization problems. Whether in orbital spacecraft formation reconfiguration [17] or in target detection task [18], PIO has made a better performance than other well-known rivals, such as Particle Swarm Optimization (PSO), Genetic Algorithm (GA) and Artificial Bee Colony (ABC) optimization. It is undeniable that PIO has some advantages in some mono-objective optimization problems. However, PIO always shrinks back at the sight of multi-objective optimization problems. Inspired by the ideas in refs. [8,19], a variant of PIO named Multi-objective Pigeon-Inspired Optimization (MPIO) (shown in Figure 1) is proposed in order to widen application range and fields of PIO. In addition, a transition factor is employed to complete a stable work transition between two operators.

The thoughts of MPIO are employed in electromagnetic fields to design the parameters of the BLDC motors. From the comparative results against NSGA-II, the symbolic multi-objective optimization algorithm, MPIO has certain superiority in uniformity and extent of the Pareto front in the face of this type of multi-objective optimization problems.

The rest of the paper is organized as follows. Section 2 demonstrates the principles of PIO. Section 3 describes the design of MPIO. Section 4 formulates the design problem for the BLDC motors. Simulation validation, together with comparison against NSGA-II, is presented in Section 5, and our concluding remarks are contained in Section 6.

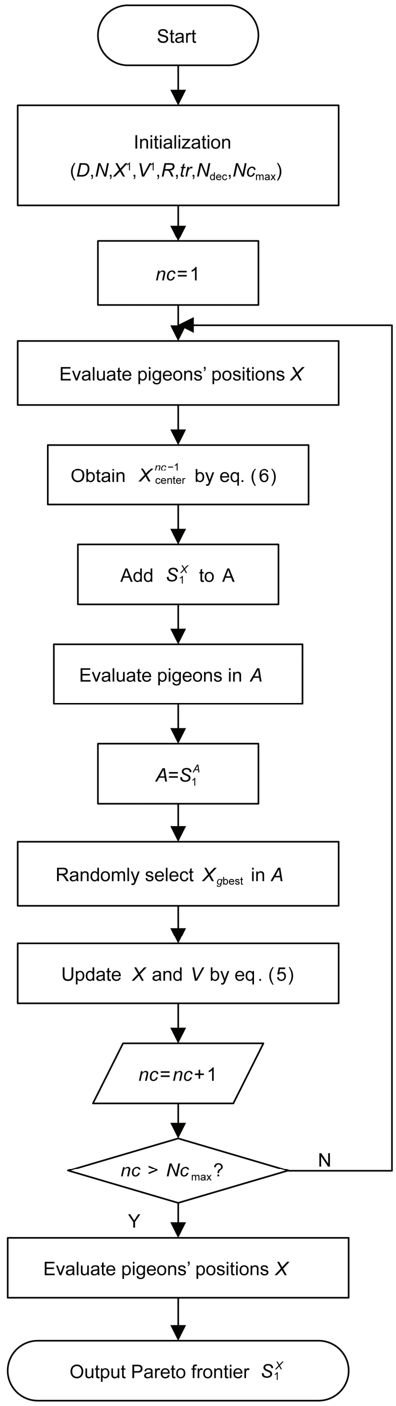

Figure 1 The detailed flow chart of MPIO.

2 Basic PIO

As presented in ref. [13], PIO adopts two operators to imitate the behavior of homing pigeons, map and compass operator, and landmark operator, respectively.

First, map and compass operator will proceed as follows. Pigeons are randomly initialized within a D-dimension search space. The total number of pigeons is N. Their positions and velocities respectively are denoted as $X_i=[x_{i1}, x_{i2},\ldots,x_{iD}]$ and $V_i=[v_{i1,}v_{i2},\ldots,v_{iD}]$, where $i=1,2,\ldots,N$. The new positions X_i and velocities V_i at next iteration are updated as follows:

$$V_i^{nc}=V_i^{nc-1}\cdot e^{-R\times nc}+\text{rand}\cdot(X_{g\text{best}}-X_i^{nc-1}),$$
$$X_i^{nc}=X_i^{nc-1}+V_i^{nc}, \qquad (1)$$

where R is the map and compass factor which is set to be between 0 and 1, nc is the current times of iterations, $X_{g\text{best}}$ is the global best position that is located by comparing all the pigeons' positions after $nc-1$ iteration cycles.

The above iteration goes ahead until terminal requirement is met [20]. The map and compass operator will hand pigeons' positions X_i over to the landmark operator.

The landmark operator will halve the total number of pigeons N in every generation. The pigeons in the lower half of the line sorted by fitness values are abandoned, because they are deemed to be far from the destination and unfamiliar with the landmarks. Then the center of pigeons' positions X_{center} is regard as the destination that every pigeon will fly to. The positions X_i are generated according to eq. (2):

$$X_{\text{center}}^{nc-1} = \frac{\sum_{i=1}^{N^{nc-1}} X_i^{nc-1} \cdot F(X_i^{nc-1})}{N^{nc-1} \cdot \sum_{i=1}^{N^{nc-1}} F(X_i^{nc-1})},$$
$$N^{nc} = \frac{N^{nc-1}}{2}, \tag{2}$$
$$X_i^{nc} = X_i^{nc-1} + \text{rand} \cdot (X_{\text{center}}^{nc-1} - X_i^{nc-1}),$$

where

$$F(X_i^{nc-1}) = \begin{cases} \dfrac{1}{\text{fitness}\left(X_i^{nc-1}\right) + \varepsilon}, & \text{for minimization problem,} \\ \text{fitness}\left(X_i^{nc-1}\right), & \text{for maximization problem.} \end{cases}$$

After $Nc_{\max}$ iterations are completed, the landmark operator will stop.

3 MPIO

3.1 Pareto sorting scheme

Pareto sorting scheme has been involved in many algorithms to deal with multi-objective optimization problems, such as NSGA-II [19, 21–23], MOPSO [8] and BA [6]. In NSGA-II, the initial population evolves to obtain a population of genotypes and a set of the phenotypes of these genotypes [24].

Due to its good performance in balancing exploitation and exploration, MPIO also adopts the elitist structure to judge the quality of the pigeon individual. The implementation of the sorting scheme is based on the following step:

1) Non-dominated sorting operator

The position of i pigeon X_i is said to dominate the position of j pigeon X_j, if and only if both of the following conditions are satisfied [6]

$$\begin{cases} f_k(X_i) \leqslant f_k(X_j), & \text{for all } k = 1, 2, \ldots, n, \\ f_{\bar{k}}(X_i) < f_{\bar{k}}(X_j), & \text{for at least one } \bar{k} \in \{1, 2, \ldots, n\}, \end{cases} \tag{3}$$

where $f_k : R^D \to R$ is the k^{th} function of the n objective functions. As a note, eq. (3) is for minimization problems. For maximization, the greater f_k is better.

Pigeons will be divided into different sets by the non-dominated sorting operator, S_1^X, S_2^X and so on (shown in Figure 2). The surface formed by solutions in best non-dominated set S_1^X is known as Pareto frontier.

2) Crowded-comparison operator

After all the pigeons have been sorted into m sets, crowded-comparison operator continues to rank the pigeons in each set by comparing the crowding distance of individual position. The crowding distance of the i pigeon in set S_j^X is defined as

$$\text{Dis}(X_i) = \frac{f_k(X_{i+1}) - f_k(X_{i-1})}{f_k^{\max} - f_k^{\min}}, \tag{4}$$

where $i \in \{2, 3, \ldots, n_j^X - 1\}$, n_j^X is the number of pigeons in set S_j^X. The maximum and minimum of the k^{th} objective function are $f_k^{\max}$ and $f_k^{\min}$, respectively. $\text{Dis}(X_1)$ and $\text{Dis}(X_{n_j^X})$ are set to be ∞. To ensure the diversity of solutions, the greater crowding distance is deemed to be better.

Multi-objective optimization problem has access to the rank of solutions X^* by Pareto sorting scheme other than comparing fitness values in mono-objective optimization. As shown in Figure 3, pigeons will go through two operations. In the first place, the non-dominated sorting operator will divide them into different sets (S_1^X, S_2^X, etc.). The crowded-comparison operator continues to rank the pigeons in each set. As a result, a sequence of pigeons in descending order is produced by Pareto sorting scheme.

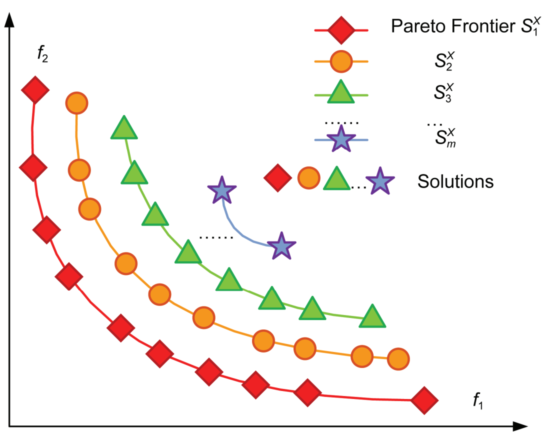

Figure 2 (Color online) Sketch of sets divided by non-dominated sorting operator.

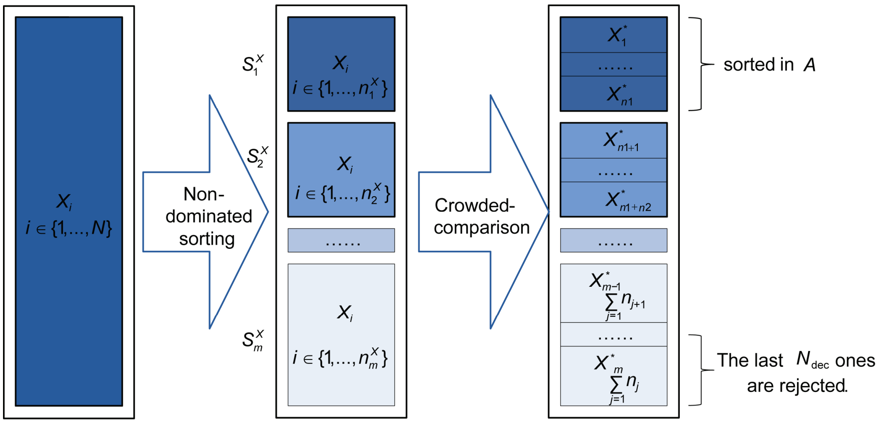

Figure 3 (Color online) Pareto sorting structure implemented in MPIO.

3.2 Consolidation operator

PIO algorithm employs two independent refinement cycles to simulate the characteristics of homing pigeons. In this paper, the map and compass operator will merge with the landmark operator for the navigation of homing pigeons (shown in Figure 4). Their specific work is shown below:

$$
\begin{aligned}
&N^{nc} = N^{nc-1} - N_{\text{dec}},\\
&V_i^{nc} = V_i^{nc-1}\cdot e^{-R\times nc}\\
&\quad +\text{rand}_1\cdot tr\cdot(1-\lg^{nc}_{Nc_{\max}})\cdot(X_{g\text{best}} - X_i^{nc-1})\\
&\quad +\text{rand}_2\cdot tr\cdot \lg^{nc}_{Nc_{\max}}\cdot(X^{nc-1}_{\text{center}} - X_i^{nc-1}),\\
&X_i^{nc} = X_i^{nc-1} + V_i^{nc},
\end{aligned}
\tag{5}
$$

where N_{dec} is the number of pigeons rejected in each iteration (shown in Figure 3), $Nc_{\max}$ is the maximum times of iterations, tr is the transition factor. As nc increased, X_i^{nc} relies more on X^{nc-1}_{center} rather than $X_{g\text{best}}$. Under the action of tr, the work transition between the two operators is completed smoothly.

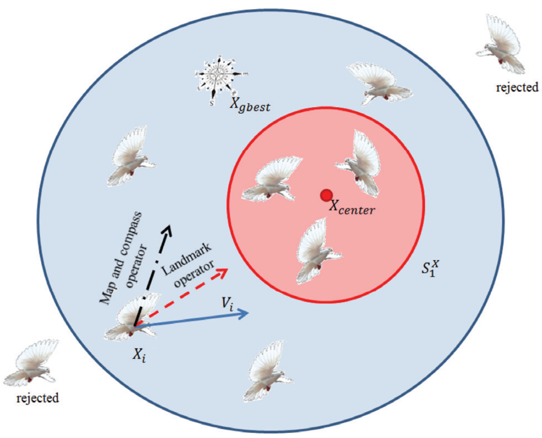

Figure 4 (Color online) Illustration of MPIO.

In multi-objective optimization, $X_{g\text{best}}$ and X^{nc-1}_{center} need to be redefined. Inspired by the thought in ref. [8], an archive A is used to store the non-dominated solutions and solve $X_{g\text{best}}$ and X^{nc-1}_{center}. The implementation is presented in the following exposition.

Step 1. After X are sorted in X^* in every generation, the pigeons in S_1^X will be added in an archive A (shown in Figure 3) and X^{nc-1}_{center} can be obtained by the following equation:

$$
X^{nc-1}_{\text{center}} = \frac{\sum_{j=1}^{n_1^X} S^X_{1j}}{n_1^X}. \tag{6}
$$

Step 2. Pareto sorting scheme is used to evaluate the fitness of each pigeons in A and thus determine which pigeons should be selected to store in the archive of the non-dominated solutions [8]. The archive A absorbs superior current non-dominated solutions in S_1^A and eliminates inferior solutions in other sets.

Step 3. Randomly select a pigeon in A as $X_{g\text{best}}$.

3.3 Procedure of MPIO

The flow chart of the optimization algorithm is described in Figure 1. The implementation of our proposed MPIO algorithm is as follows:

Step 1. Initialize pigeons randomly with positions X^1 and velocities V^1. The maximum number of iterations is assigned as $Nc_{\max}$.

Step 2. Evaluate the pigeons' positions X by Pareto sorting scheme. Obtain X^{nc-1}_{center} by eq. (6) and add the pigeons in S_1^X to an archive A.

Step 3. Pareto sorting scheme is also used to evaluate the pigeons in A. Only the non-dominated solutions will be kept in *A*, that is $A = S_1^A$.

Step 4. Randomly select a pigeon in *A* as X_{gbest}.

Step 5. Update velocities *V* and positions *X* according to eq. (5).

Step 6. Update the iteration counter *nc* by $nc = nc + 1$.

Step 7. If *nc* is less than $Nc_{\max}$, return to Step 2. Otherwise evaluate the current positions *X* and output the Pareto frontier S_1^X.

3.4 Theoretical analysis

The necessary theoretical analysis is conducted to illustrate the rationality and performance of MPIO.

1) Convergence analysis

In summary, MPIO mainly contains two parts: Pareto sorting scheme in Section 3.1 and status update by operators in Section 3.2. In ref. [25], Xie and Ding have already demonstrated that some multi-objective evolutionary algorithms with Pareto sorting scheme converge to the global optimality of multi-objective problem with probability one. Therefore, the convergence analysis of MPIO can be transformed into the convergence proof of the consolidation operator in eq. (5).

Although N_{dec} pigeons fade away in every iteration, $N - N_{\text{dec}} \cdot (Nc_{\max} - 1)$ original pigeons finally remain, which also indicates the convergence property of consolidation operator. Eq. (5) can be rewritten as

$$\begin{aligned} V_i^{nc} &= e^{-R\times nc} \cdot V_i^{nc-1} \\ &\quad - tr \cdot [(\text{rand}_2 - \text{rand}_1) \cdot \lg_{Nc_{\max}}^{nc} + \text{rand}_1] \cdot X_i^{nc-1} \\ &\quad + \text{rand}_1 \cdot tr \cdot (1 - \lg_{Nc_{\max}}^{nc}) \cdot X_{g\text{best}} \\ &\quad + \text{rand}_2 \cdot tr \cdot \lg_{Nc_{\max}}^{nc} \cdot X_{\text{center}}^{nc-1}, \\ X_i^{nc} &= e^{-R\times nc} \cdot V_i^{nc-1} \\ &\quad + \left\{1 - tr \cdot [(\text{rand}_2 - \text{rand}_1) \cdot \lg_{Nc_{\max}}^{nc} + \text{rand}_1]\right\} \cdot X_i^{nc-1} \\ &\quad + \text{rand}_1 \cdot tr \cdot (1 - \lg_{Nc_{\max}}^{nc}) \cdot X_{g\text{best}} \\ &\quad + \text{rand}_2 \cdot tr \cdot \lg_{Nc_{\max}}^{nc} \cdot X_{\text{center}}^{nc-1}. \end{aligned} \tag{7}$$

Eq. (7) can be recast in a matrix form:

$$\begin{aligned} \begin{bmatrix} V^{nc} \\ X^{nc} \end{bmatrix} &= \\ &\begin{bmatrix} e^{-R\times nc} & -tr \cdot [(\text{rand}_2 - \text{rand}_1) \cdot \lg_{Nc_{\max}}^{nc} + \text{rand}_1] \\ e^{-R\times nc} & 1 - tr \cdot [(\text{rand}_2 - \text{rand}_1) \cdot \lg_{Nc_{\max}}^{nc} + \text{rand}_1] \end{bmatrix} \begin{bmatrix} V^{nc-1} \\ X^{nc-1} \end{bmatrix} \\ &+ \begin{bmatrix} \text{rand}_1 \cdot tr \cdot (1 - \lg_{Nc_{\max}}^{nc}) & \text{rand}_2 \cdot tr \cdot \lg_{Nc_{\max}}^{nc} \\ \text{rand}_1 \cdot tr \cdot (1 - \lg_{Nc_{\max}}^{nc}) & \text{rand}_2 \cdot tr \cdot \lg_{Nc_{\max}}^{nc} \end{bmatrix} \begin{bmatrix} X_{g\text{best}} \\ X_{\text{center}}^{nc-1} \end{bmatrix}. \end{aligned} \tag{8}$$

For simplicity, eq. (8) can be expressed as

$$\begin{bmatrix} V^{nc} \\ X^{nc} \end{bmatrix} = G \begin{bmatrix} V^{nc-1} \\ X^{nc-1} \end{bmatrix} + d, \tag{9}$$

where

$$d = \begin{bmatrix} \text{rand}_1 \cdot tr \cdot (1 - \lg_{Nc_{\max}}^{nc}) & \text{rand}_2 \cdot tr \cdot \lg_{Nc_{\max}}^{nc} \\ \text{rand}_1 \cdot tr \cdot (1 - \lg_{Nc_{\max}}^{nc}) & \text{rand}_2 \cdot tr \cdot \lg_{Nc_{\max}}^{nc} \end{bmatrix} \begin{bmatrix} X_{g\text{best}} \\ X_{\text{center}}^{nc-1} \end{bmatrix},$$

$$G = \begin{bmatrix} \omega & \varphi \\ \omega & 1+\varphi \end{bmatrix}, \quad \varphi = -tr \cdot \left[(\text{rand}_2 - \text{rand}_1) \cdot \lg_{Nc_{\max}}^{nc} + \text{rand}_1\right]$$

and $\omega = e^{-R\times nc}$.

For any given $\begin{bmatrix} V^1 \\ X^1 \end{bmatrix}$, a sufficient and necessary convergent condition for eq. (9) is that the spectral radius of *G* is less than 1.

The characteristic polynomial of G is shown as

$$\lambda^2 - (1 + \omega + \varphi)\lambda + w = 0. \tag{10}$$

The spectral radius of G can be obtained.

$$\rho(G) = \max\left\{|\lambda_1|, |\lambda_2|\right\}, \tag{11}$$

where λ_1 and λ_2 are the eigenvalues of *G*.

A sufficient and necessary condition for the case that all the roots of eq. (10) are modulo less than 1 is that the following inequalities are satisfied simultaneously:

$$\begin{cases} |\omega| < 1, \\ \omega + [-(1+\omega+\varphi)] + 1 > 0, \\ \omega - [-(1+\omega+\varphi)] + 1 > 0. \end{cases} \tag{12}$$

In conclusion, a necessary convergent condition for the consolidation operator in eq. (5) is that the following inequalities are satisfied simultaneously:

$$\begin{cases} e^{-R\times nc} < 1, \\ 0 < tr \cdot [(\text{rand}_2 - \text{rand}_1) \cdot \lg_{Nc_{\max}}^{nc} + \text{rand}_1] < 2 + 2 \cdot e^{-R\times nc}. \end{cases} \tag{13}$$

A more strict necessary convergent condition can be simplified as

$$\begin{cases} R > 0, \\ 0 < tr \leqslant 1. \end{cases} \tag{14}$$

2) Time complexity analysis

An attempt was finally made towards a systematical time complexity analysis that turns the theory of evolutionary algorithms into "a legal part of the theory of efficient algorithms" [26]. A brief analysis of time complexity of MPIO is carried out to provide the efficiency quantitatively.

Pareto sorting scheme contains two parts: non-dominated sorting operator and crowded-comparison operator. The time complexity of non-dominated sorting operator is $O(PQ^2)$ in each execution, while the other operator is

$O(PQ\log Q)$, where P is the number of the objective functions and Q is the number of individuals involved in the calculation. It is obvious that the main source of time complexity in Pareto sorting scheme is non-dominated sorting operator. It means that the time complexity of Pareto sorting scheme is $O(PQ^2)$.

The time complexity analysis of main operations in MPIO during every iteration is given in Table 1. It is evident that the time complexity of MPIO mainly lies in evaluating pigeons' positions X. In other words, the major affecting factor of time complexity in MPIO is Pareto sorting scheme, which is the same as other multi-objective evolutionary algorithms.

Therefore, the time complexity of MPIO is shown as

$$T=\sum_{nc=1}^{Nc_{\max}} \mathrm{O}(n(N^{nc})^2)=\sum_{nc=1}^{Nc_{\max}} O(n(N-N_{\mathrm{dec}}(nc-1))^2)$$
$$=O(n(Nc_{\max}N(N-N_{\mathrm{dec}}(Nc_{\max}-1))$$
$$+\frac{Nc_{\max}(Nc_{\max}-1)(2Nc_{\max}-1)}{6}N_{\mathrm{dec}}{}^2)). \quad (15)$$

4 BLDC motor parameter design

4.1 Analytical model of BLDC motor

In this paper, the optimization for an analytical model of the BLDC motors is chosen as the benchmark to test the feasibility of MPIO [20]. The analytical model was proposed in ref. [27], and its mathematical model can be completely depicted in ref. [28]. It is composed of 78 nonlinear equations implemented with 5 design variables and 6 constraints to optimize in mono-objective case, or with 5 constraints in multi-objective case [6].

This study focuses on the multi-objective case, while adopts the novel idea of PIO to electromagnetic fields. As shown in Table 2, the optimization is aimed at obtaining the maximum efficiency $\eta_{\max}$ and the minimum total mass $M_{\mathrm{tot_{min}}}$ within the allowable range of design variables under the limit of the constraint variables. It can be expressed as the following multi-objective optimization problem:

$$\text{Minimize}F=[(1-\eta),M_{\mathrm{tot}}],$$
$$\text{with}\begin{cases}150\text{ mm}<D_s<330\text{ mm},\\ 0.9\text{ T}<B_d<1.8\text{ T},\\ 2.0(\mathrm{A/mm^2})<\delta<5.0(\mathrm{A/mm^2}),\\ 0.5\text{ T}<B_e<0.76\text{ T},\\ 0.6\text{ T}<B_{cs}<1.6\text{ T s.t.},\\ D_{\mathrm{ext}}<340\text{ mm},\\ D_{\mathrm{int}}>76\text{ mm},\\ I_{\max}>125\text{ A},\\ \mathrm{discr}(D_s,\delta,B_d,B_e)>0,\\ T<120°\mathrm{C}.\end{cases} \quad (16)$$

Table 1 Time complexity analysis of main operations in MPIO

Operation	Time complexity
Evaluate pigeons' positions X	$O(n(N^{nc})^2)$
Obtain by $X_{\mathrm{center}}^{nc-1}$ eq. (6)	$O(n_1^X)$
Evaluate pigeons in A	$O(n(n_1^A)^2)$
Update X and V by eq. (5)	$O(N^{nc})$

Table 2 BLDC motor model parameters

	Variables	Description	Value
Objective variables	$\eta(\%)$	efficiency	–
	M_{tot}(kg)	total mass	–
Design variables	D_s(mm)	bore stator diameter	(150,330)
	B_d(T)	magnetic induction in the teeth	(0.9,1.8)
	$\delta(\mathrm{A/mm^2})$	current density in the conductors	(2.0,5.0)
	B_e(T)	magnetic induction in the air gap	(0.5,0.76)
	B_{cs}(T)	magnetic induction in the back iron	(0.6,1.6)
Constraint variables	D_{ext}(mm)	outer diameter	$(-\infty,340)$
	D_{int}(mm)	inner diameter	$(76,\infty)$
	$I_{\max}$(A)	magnetics maximum current	$(125,\infty)$
	$\mathrm{discr}(D_s,\delta,B_d,B_e)$	determinant used in the calculation	$(0,\infty)$
	T_a(°C)	temperature	$(-\infty,120)$

4.2 Details in parameter design based on MPIO

The process of the BLDC motor parameter design based on the MPIO algorithm is nearly the same as the implementation in Section 3.3. There are two differences worth reiterating.

In Step 1, the initialization is as follows:

1) The dimension D of search space for positions X_i and velocities V_i is 5.

2) $X_i=[x_{i1},x_{i2},x_{i3},x_{i4},x_{i5}]=[D_s,B_d,\delta,B_e,B_{cs}]$ and x_{i1} ,.., x_{i5} are within the corresponding value range.

3) Initialize V^1 randomly.

4) The other parameters (N, R, tr, N_{dec} and $Nc_{\max}$) can be adjusted according to the performance of MPIO.

In addition, the non-dominated sorting operator divides pigeons into different sets according to two functions:

$$\begin{cases}f_1(X_i)=1-\eta,\\ f_2(X_i)=M_{\mathrm{tot}}.\end{cases} \quad (17)$$

The BLDC motor parameter problem is a minimization problem. The less f_1 and f_2 are better.

The final Pareto frontier output contains X_i, $f_1(X_i)$ and $f_2(X_i)$ in S_1^X. $f_1(X_i)$ and $f_2(X_i)$ can be described in figure to evaluate the performance of MPIO. X_i is the BLDC motor parameters designed by MPIO.

5 Simulation results

In this paper, the simulation verification consists of two parts. Before the multi-objective optimization for a model of the BLDC motors is chosen as the benchmark, the function KUR, proposed by Kursawe in ref. [29], will be analyzed firstly. As given in eq. (18), the benchmark optimization problem has two objectives to be minimized.

$$\begin{cases} f_1(x)=\sum_{i=1}^{n-1}(-10e^{(-0.2)\sqrt{x_i^2+x_{i+1}^2}}), \\ f_2(x)=\sum_{i=1}^{n}(|x_i|^{0.8}+5\sin(x_i)^3), \end{cases} \tag{18}$$

where $n=3$ and $x_i\in[-5,5]$.

Both algorithms run 10 times for the multi-objective problem with parameters in Table 3. To ensure fair and valid comparisons, the algorithms involved in the comparison should have the same amount of calculation and the same number of final solutions. Eq. (19) is supposed to be satisfied on the selection of the parameters in algorithms.

$$N\cdot Nc_{\max}-\frac{Nc_{\max}^2-Nc_{\max}}{2}\cdot N_{\text{dec}}=\text{Num}\cdot \text{Itr}_{\max}, \tag{19}$$

where Num and $\text{Itr}_{\max}$ are the size of population and the maximum number of iterations in NSGA-II, respectively. In this benchmark, we choose 298 pigeons to find solutions at the beginning of 100 iterations with 2 pigeons rejected in every generation due to unfamiliarity with the landmarks. Accordingly, Num and $\text{Itr}_{\max}$ are set to be 100 and 199.

Pareto frontiers of the 10 simulations are also operated by Pareto sorting scheme. The finally obtained Pareto frontier (shown in Figure 5) contains the optimal solutions (shown in Table 4) of 10 simulations. From Table 4 and Figure 5, the minimum value of objective f_1 found by MPIO is slightly more than that by NSGA-II, however, in the minimum value of f_2 and Pareto frontier of solutions, MPIO presents a satisfactory performance.

To properly evaluate MPIO performance, three important criteria in ref. [30] are considered when compared with NSGA-II. The D-metric provides an indication of the accuracy of the approximation computed in terms of the maximum distance with the dominated points [31], which is defined between 0 and 1, with smaller values indicating better accuracy. The Δ-metric computes the average distance between each pair of consecutive points, the smaller value of which is an indication of better uniformity. It is also defined to be between 0 and 1. The ∇-metric describes the extent of the Pareto front by calculating the difference between the maximum and minimum values for the different objectives. Larger values are expected for a better extent. After normalization of objective vectors for an independent calculations scale, the statistical results of 10 simulations for three performance criteria are obtained. From Table 5, MPIO has better accuracy and uniformity than NSGA-II. Equally important, the mean Pareto front extent of MPIO is in a lack of breadth. From the analysis of the standard deviation data, we can see that MPIO can give an amazing performance as shown in Figure 5 with better accuracy, well distributed solutions and a wide range for each objective function value, but it also presents a poorer consistency in uniformity and

Table 3 Parameters of MPIO and NSGA-II

Algorithm	Parameters	Description	Value
MPIO	N	size of pigeons	298
	$Nc_{\max}$	maximum times of iterations	100
	N_{dec}	the number of pigeons rejected in each iteration	2
	R	the map and compass factor	0.3
	tr	the transition factor	1
NSGA-II	Num	size of population	100
	$\text{Itr}_{\max}$	maximum times of iterations	199
	S_{pool}	size of a mating pool after tournament selection	25
	S_{tour}	size of the tournament	2
	η_c	crossover distribution index	20
	η_m	mutation distribution index	20

Table 4 Solutions for KUR by MPIO and NSGA-II

Mini-objective		f_1		f_2	
Algorithm		MPIO	NSGA-II	MPIO	NSGA-II
Objectives	f_1	−19.9910	−19.9961	−13.8775	−13.5339
	f_2	0.0115	0.0083	−11.2403	−11.0193
Design variables	x_1	8.4554×10^{-5}	0.0013	−1.1524	−1.1527
	x_2	−0.0022	4.8531×10^{-4}	−1.1525	−1.1531
	x_3	8.7649×10^{-4}	-1.9510×10^{-4}	1.6747	−1.9868

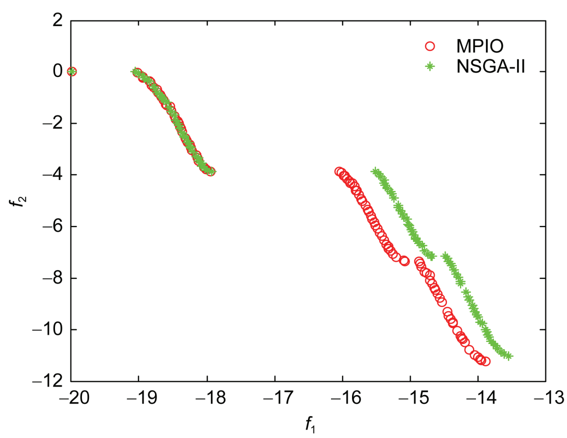

Figure 5 (Color online) Pareto frontier of MPIO and NSGA-II for KUR.

Table 5 Performance criteria for KUR by MPIO and NSGA-II

Algorithm	D-metric		Δ-metric		∇-metric	
	average	std. dev.	average	std. dev.	average	std. dev.
MPIO	0.7614	0.0079	0.0559	0.0253	54.6374	26.3498
NSGA-II	0.7802	0.0208	0.0666	0.0155	71.4828	4.6624

extent of the Pareto front than NSGA-II.

To further test the effectiveness of MPIO, it is applied to the design of a BLDC motor described in Section 4. Both algorithms also run 10 times for the multi-objective problem. The final Pareto frontier is also gained from Pareto frontiers of 10 simulations by Pareto sorting scheme. In order to present and compare the Pareto frontier of two algorithms clearly, we select fewer pigeons and less population in the simulation. Under the condition of eq. (19), the size of pigeons N, the maximum times of MPIO iteration $Nc_{\max}$, the size of population Num and the maximum times of NSGA-II iteration $\mathrm{Itr}_{\max}$ are set to be 148, 50, 50 and 99, respectively. The other parameters are the same as Table 3. From Figure 6 and Table 6, MPIO gives comparable performance with NSGA-II, even better than NSGA-II. The most of Pareto frontier of MPIO is surrounded by outside NSGA-II and it does embody certain superiority in uniformity and extent of the Pareto front. However, the lack of consistency for MPIO is also verified in the BLDC motor design.

6 Conclusion

PIO is a newly invented swarm intelligence optimization algorithm, which is inspired by the behaviors of homing pigeons. Map and compass operator model and landmark operator model are presented based on the pigeons' homing tools, magnetic field and sun, and landmarks, respectively.

In this paper, inspired by the Pareto sorting scheme in ref. [19] and archive A stored with non-dominated solutions in

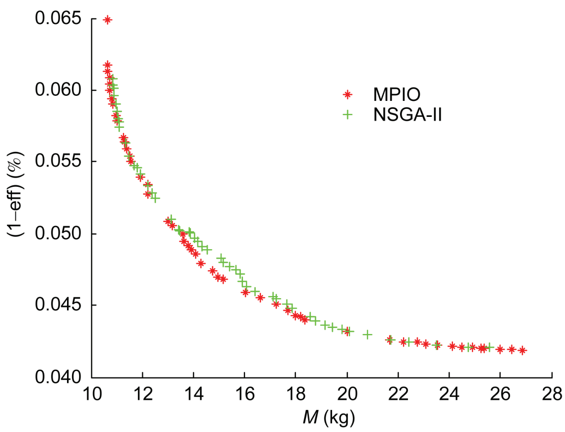

Figure 6 (Color online) Pareto frontier of MPIO and NSGA-II for BLDC motors design.

Table 6 Solutions for BLDC motor design by MPIO and NSGA-II

Mini-objective	Type of variables	Variables	MPIO	NSGA-II
$1-\eta$	Objective variables	$(1-\eta)_{\min}(\%)$	0.0419	0.0421
		$M_{\mathrm{tot}}(\mathrm{kg})$	26.8513	25.5709
	Design variables	$D_s(\mathrm{mm})$	271.2335	265.9730
		$B_d(\mathrm{T})$	1.2617	1.3540
		$\delta(\mathrm{A/mm^2})$	2.0000	2.0000
		$B_e(\mathrm{T})$	0.7003	0.6964
		$B_{cs}(\mathrm{T})$	0.6000	0.6000
M_{tot}	Objective variables	$(1-\eta)(\%)$	0.0649	0.0608
		$M_{\mathrm{tot_{min}}}(\mathrm{kg})$	10.6226	10.8451
	Design variables	$D_s(\mathrm{mm})$	187.7795	181.2067
		$B_d(\mathrm{T})$	1.7982	1.8000
		$\delta(\mathrm{A/mm^2})$	3.7525	3.4087
		$B_e(\mathrm{T})$	0.6618	0.6607
		$B_{cs}(\mathrm{T})$	1.6000	1.3027

ref. [8], a variant of PIO named MPIO is proposed to solve multi-objective optimization problems. In addition, a transition factor tr is employed to complete a stable work transition between two operators. The thoughts of MPIO are employed in electromagnetic field to design the parameters of the BLDC motors. From the comparative results against the NSGA-II, MPIO has certain superiority in solving multi-objective optimization problems.

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