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Linear-quadratic regulator controller design for quadrotor based on pigeon-inspired optimization

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Abstract

Purpose – The purpose of this paper is to propose a new algorithm for linear-quadratic regulator (LQR) controller of a quadrotor with fast and stable performance, which is based on pigeon-inspired optimization (PIO).

Design/methodology/approach – The controller is based on LQR. The determinate parameters are optimized by PIO, which is a newly proposed swarm intelligent algorithm inspired by the characteristics of homing pigeons.

Findings – The PIO-optimized LQR controller can obtain the optimized parameters and achieve stabilization in about 3 s.

Practical implications – The PIO-optimized LQR controller can be easily applied to the flight formation, autonomous aerial refueling (AAR) and detection of unmanned aerial vehicles, especially applied to (AAR) in this paper.

Originality/value – This research applies PIO to optimize the tuning parameters of LQR, which can considerably improve the fast and stabilizing performance of attitude control. The simulation results show the effectiveness of the proposed algorithm.

Keywords Quadrotor, Dynamic model, Pigeon-inspired optimization, Linear-quadratic regulator

Paper type Research paper

Introduction

In the control system of unmanned aerial vehicles (UAVs), the attitude control is a foundation. One of the challenging problems for attitude control of UAVs is to design an optimized robust control system with fast, accuracy and stabilized responses. Researchers put forward many control techniques, such as proportion-integration-differentiation control, linear-quadratic regulator (LQR) control and so on. LQR can realize the control targets with the minimum energy cost. In this paper, we establish the model of quadrotor and apply LQR to control the attitude of the quadrotor. The tuning parameters of LQR are optimized by pigeon-inspired optimization (PIO) algorithm, whose performance is fast and stable.

In recent years, there are a boom of the bio-inspired optimization algorithms, which are derived from biological inspired self-organized systems such as ants, bees, pigeons and so on. The genetic algorithm was firstly put forward by Holland (1973) to study the self adaptation behavior of natural system. Particle swarm optimization (PSO) was developed by Kennedy and Eberhart (1995), which were inspired by social behavior of bird flocking or fish schooling. Ant colony optimization was proposed by Colorni *et al.* (1991) under the inspiration of the collective behavior on real ant

system. Artificial bee colony optimization was a bio-inspired optimization based on the intelligent foraging behavior of a honey bee swarm, proposed by Karaboga (2005). Moreover, there appeared many other algorithms including biogeography-based optimization (Wang *et al.*, 2013), brain storm optimization (Sun *et al.*, 2013) and so on. In this paper, we adopt the PIO algorithm, which was first put forward by Duan *et al.* (2014), inspired by the homing characteristics of pigeons. The PIO algorithm has been verified for its efficiency and robustness in Duan *et al.* (2014), which is first applied to air robot path planning. Later, Li and Duan (2014) successfully put PIO algorithm into target detection. Moreover, Zhang and Duan (2015) apply PIO to orbital spacecraft formation reconfiguration. Duan *et al.* (2015) use PIO to close formation cooperative control.

Attitude control is a critical procedure of flight control system, which needs to control three angles, including pitch angle, roll angle and yaw angle. Take the autonomous aerial refueling (AAR) for an example. With accuracy and robust attitude control, the tanker aircraft is able to refuel the receiver UAV during the docking phase. In this paper, we adopt LQR control. The LQR is an automated way of finding an appropriate state-feedback controller. It makes control system engineer easier in the procedure of optimizing the controller. However, the engineer still need to specify the weighting

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factors and compare the results with the specified design goals. Therefore, the PIO algorithm is used for parameter optimization of weighting matrix Q in LQR, which makes the attitude controller design much easier for engineers. In this paper, the hybrid approach of LQR and PIO is proposed for attitude control.

The rest of this paper is organized as follows. The second section introduces the mathematical model of the quadrotor. The PIO algorithm is introduced in the third section. The hybrid approach of LQR and PIO is given in the fourth section. In the following section, the simulation and comparison results are presented. The sixth section concludes the paper and future work is shown.

Mathematical model of the quadrotor

The first procedure to design a LQR controller is to establish a mathematical model of quadrotors.

Coordinate transformation

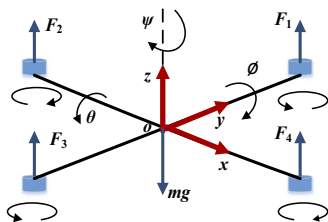
The quadrotor in experimental environment consists of four fixed-pitch rotors mounted at the four ends of a simple cross frame, as shown in Figure 1. There are two kinds of coordinate system, including body-fixed coordinate system $oxyz$ (Figure 2) and ground-fixed coordinate system $OXYZ$.

where ϕ , θ , ψ are, respectively, roll angle, pitch angle and yaw angle, which are the main control variables for attitude control. F_1, F_2, F_3, F_4 are the forces, which are generated from the four motors. mg is the weight of the quadrotor, and $oxyz$ is the body-fixed coordinate system.

Figure 1 The quadrotor



Figure 2 The quadrotor physical model and body-fixed coordinate



It is easy for us to get the lifting forces and torques in the body-fixed coordinate ($oxyz$). However, we need to analysis the force situation of quadrotor in the ground-fixed coordinate ($OXYZ$). Therefore, coordinate transformation (Figure 3) (Wang, 2014) is necessary.

We can deduce the transform matrix of every axis from $oxyz$ to $OXYZ$, as follows:

$$R(x, \phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \quad (1)$$

$$R(y, \theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \quad (2)$$

$$R(z, \psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Therefore, the transform matrix from body-fixed coordinate ($oxyz$) to ground-fixed coordinate ($OXYZ$) is as follows:

$$R(\phi, \theta, \psi) = R(z, \psi)R(y, \theta)R(x, \phi) \quad (4)$$

$$R(\phi, \theta, \psi) = \begin{bmatrix} \cos \theta \cos \psi & \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ \cos \theta \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \quad (5)$$

Mathematical model

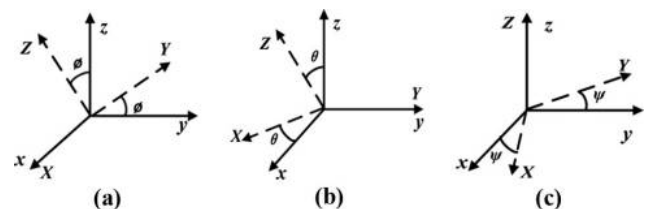
To simplify the model of quadrotor, we make some assumptions as follows:

- the structure is considered to be rigid and strictly symmetrical;
- the body-fixed coordinate origin is supposed to coincide with the center of mass;
- there is a proportional relationship between the DC voltage and torque;
- the gyroscopic effect and air resistance can be ignored when flying at low speed; and
- the attitude change is very small (it is generally considered to be less than 5°).

The lifting forces from the four rotors are, respectively, F_1, F_2, F_3 and F_4 . The lifting forces in body-fixed coordinate ($oxyz$) are as follows:

$$F_B = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix} \quad (6)$$

Figure 3 Schematic diagram of attitude



Notes: (a) The roll angle; (b) the pitch angle; (c) the yaw angle

The lifting forces need to transform from the body-fixed coordinate to the ground-fixed coordinate.

$$F_G = \begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} = R(\phi, \theta, \psi) \cdot F_B = \begin{pmatrix} \sum_{i=1}^4 F_i \\ \sum_{i=1}^4 F_i \\ \sum_{i=1}^4 F_i \end{pmatrix} \begin{bmatrix} \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ \cos \theta \cos \phi \end{bmatrix} \quad (7)$$

According to the equations of motion, we can deduce the following equation (8):

$$\begin{cases} \ddot{x} = F_X/m \\ \ddot{y} = F_Y/m \\ \ddot{z} = F_Z/m - g \end{cases} \quad (8)$$

We can deduce attitude change from Newton-Euler formula:

$$\mathcal{J}\dot{\omega} + \omega \times \mathcal{J}\omega = \tau \quad (9)$$

where $\mathcal{J} = \text{diag}(\mathcal{J}_\theta, \mathcal{J}_\phi, \mathcal{J}_\psi)$, $\mathcal{J}_\theta, \mathcal{J}_\phi, \mathcal{J}_\psi$ are, respectively, equivalent moment of inertia about the pitch axis, equivalent moment of inertia about the roll axis and equivalent moment of inertia about the yaw axis; $\omega = (\dot{\theta}, \dot{\phi}, \dot{\psi})^T$; $\tau = (\tau_1, \tau_2, \tau_3)^T$, τ is the torque, which consists of the torque (M_i) generated by lifting forces and the torques (τ_i^l) generated by motors' self-rotation, as in equation (10).

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} \tau_1^l \\ \tau_2^l \\ \tau_3^l \end{pmatrix} + \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix}, \begin{cases} \tau_1^l = \mathcal{J}_\theta \Omega \dot{\phi} \\ \tau_2^l = \mathcal{J}_\phi \Omega \dot{\theta} \\ \tau_3^l = \mathcal{J}_\psi \Omega \dot{\psi} \end{cases}, \begin{cases} M_1 = l(F_3 - F_1) \\ M_2 = l(F_4 - F_2) \\ M_3 = F_2 + F_4 - F_1 - F_3 \end{cases}, \Omega = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4 \quad (10)$$

Considering equations (9) and (10), we can deduce the equation (11):

$$\begin{cases} \mathcal{J}_\theta \ddot{\phi} = [\dot{\theta} \dot{\psi} (\mathcal{J}_\phi - \mathcal{J}_\psi) + \mathcal{J}_r \dot{\theta} \Omega + l(F_4 - F_2)] \\ \mathcal{J}_\phi \ddot{\theta} = [\dot{\phi} \dot{\psi} (\mathcal{J}_\psi - \mathcal{J}_\theta) + \mathcal{J}_r \dot{\phi} \Omega + l(F_3 - F_1)] \\ \mathcal{J}_\psi \ddot{\psi} = [\dot{\phi} \dot{\theta} (\mathcal{J}_\theta - \mathcal{J}_\phi) + \mathcal{J}_r \dot{\psi} \Omega + (Q_1 - Q_2 + Q_3 - Q_4)] \end{cases} \quad (11)$$

We define the input variables of the quadrotor as the following equation (12):

$$\begin{cases} U_1 = F_1 + F_2 + F_3 + F_4 = K_l(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = F_4 - F_2 = K_t(\Omega_4^2 - \Omega_2^2) \\ U_3 = F_3 - F_1 = K_t(\Omega_3^2 - \Omega_1^2) \\ U_4 = -Q_1 + Q_2 - Q_3 + Q_4 = K_d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{cases} \quad (12)$$

where U_1 is the control variable of vertical speed; U_2 is the control variable of roll angle; U_3 is the control variable of pitch angle; U_4 is the control variable of yaw angle; Ω_i is the rotating speed of the i -th rotor; K_l is the coefficient of lifting forces; K_t is the coefficient of torques.

Considering the above equations (7)-(12), the mathematical model of quadrotor is finally presented as equation (13) (Fernando *et al.*, 2013):

$$\begin{cases} \ddot{x} = (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) U_1/m \\ \ddot{y} = (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) U_1/m \\ \ddot{z} = (\cos \theta \cos \phi) U_1/m - g \\ \ddot{\phi} = [\dot{\theta} \dot{\psi} (\mathcal{J}_\phi - \mathcal{J}_\psi) + \mathcal{J}_r \dot{\theta} \Omega + l U_2] / \mathcal{J}_\theta \\ \ddot{\theta} = [\dot{\phi} \dot{\psi} (\mathcal{J}_\psi - \mathcal{J}_\theta) + \mathcal{J}_r \dot{\phi} \Omega + l U_3] / \mathcal{J}_\phi \\ \ddot{\psi} = [\dot{\phi} \dot{\theta} (\mathcal{J}_\theta - \mathcal{J}_\phi) + \mathcal{J}_r \dot{\psi} \Omega + U_4] / \mathcal{J}_\psi \end{cases} \quad (13)$$

where $\mathcal{J}_\theta, \mathcal{J}_\phi, \mathcal{J}_\psi$ are equivalent moment of inertia; \mathcal{J}_r is equivalent moment of inertia about motors; ϕ, θ, ψ are, respectively, roll angle, pitch angle and yaw angle; l is the distance between pivot to each motor; $\ddot{x}, \ddot{y}, \ddot{z}$ are the acceleration about every axis; U_i is the equivalent input variables; Ω is the sum of the four rotors' rotating speed; m is the weight of UAV; g is the gravity acceleration.

The parameters definition and their values are given in Table I.

Characteristic analysis

Choose the state variables $X = (x_1, x_2, x_3, x_4, x_5, x_6)^T = (\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi})^T$, input of the system $U = (U_2, U_3, U_4)^T$.

According to equation (13), we can deduce the following equation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_4 x_6 (\mathcal{J}_\phi - \mathcal{J}_\psi) / \mathcal{J}_\theta + \mathcal{J}_r \dot{\theta} \Omega / \mathcal{J}_\theta + l U_2 / \mathcal{J}_\theta \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = x_2 x_6 (\mathcal{J}_\psi - \mathcal{J}_\theta) / \mathcal{J}_\phi + \mathcal{J}_r \dot{\phi} \Omega / \mathcal{J}_\phi + l U_3 / \mathcal{J}_\phi \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = x_2 x_4 (\mathcal{J}_\theta - \mathcal{J}_\phi) / \mathcal{J}_\psi + \mathcal{J}_r \dot{\psi} \Omega / \mathcal{J}_\psi + U_4 / \mathcal{J}_\psi \end{cases} \quad (14)$$

Following assumptions are used to simplify the dynamic modeling of the quadrotor.

- the cross-coupling effects of angular speeds (Coriolis-centripetal effect) and gyroscopic effects are negligible; and
- in hovering conditions, the accelerations in the body-fixed coordinate are approximately equal to the accelerations in the ground-fixed coordinate.

The state space equations of pose control system are presented as follows:

Table I Model parameters of a quadrotor

Parameter	Meaning	Value
$K_l (N \cdot s^2)$	Coefficient of lifting forces	0.0211
$K_t (N \cdot m \cdot s^2)$	Coefficient of torques	0.00684
$J_\theta (kg \cdot m^2)$	Equivalent moment of inertia about the pitch axis	0.0552
$J_\phi (kg \cdot m^2)$	Equivalent moment of inertia about the roll axis	0.0552
$J_\psi (kg \cdot m^2)$	Equivalent moment of inertia about the yaw axis	0.110
$m (kg)$	Weight of quadrotor	1.96
$l (m)$	Distance between pivot to each motor	0.200

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 1/\mathcal{J}_\theta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/\mathcal{J}_\phi & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/\mathcal{J}_\psi \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

where $1/\mathcal{J}_\theta = 3.623$; $1/\mathcal{J}_\phi = 3.623$; $1/\mathcal{J}_\psi = 9.091$.

The zero-input responses of attitudes are shown in Figure 4. As we can see from Figure 4 that in original state with bad parameters, the responses of attitudes take more than 10 s to die away and have huge amplitudes, which can not satisfy the control targets.

Pigeon-inspired optimization algorithm

Duan and Qiao (2014) first put forward the PIO algorithm motivated by the homing pigeons, which is applied to air robot path planning. Li and Duan (2014) verify its efficiency by using for target detection. Dou and Duan (2015) apply PIO to model prediction control for UAV. Compared with

conventional algorithms, PIO is a more reliable, feasible and effective method with fast convergence rate. In this paper, the determinate parameters of LQR are optimized by the PIO. The basic PIO includes two operators: map and compass operator and landmark operator. The map and compass operator model is based on magnetic field and sun, while the landmark operator model is based on landmarks.

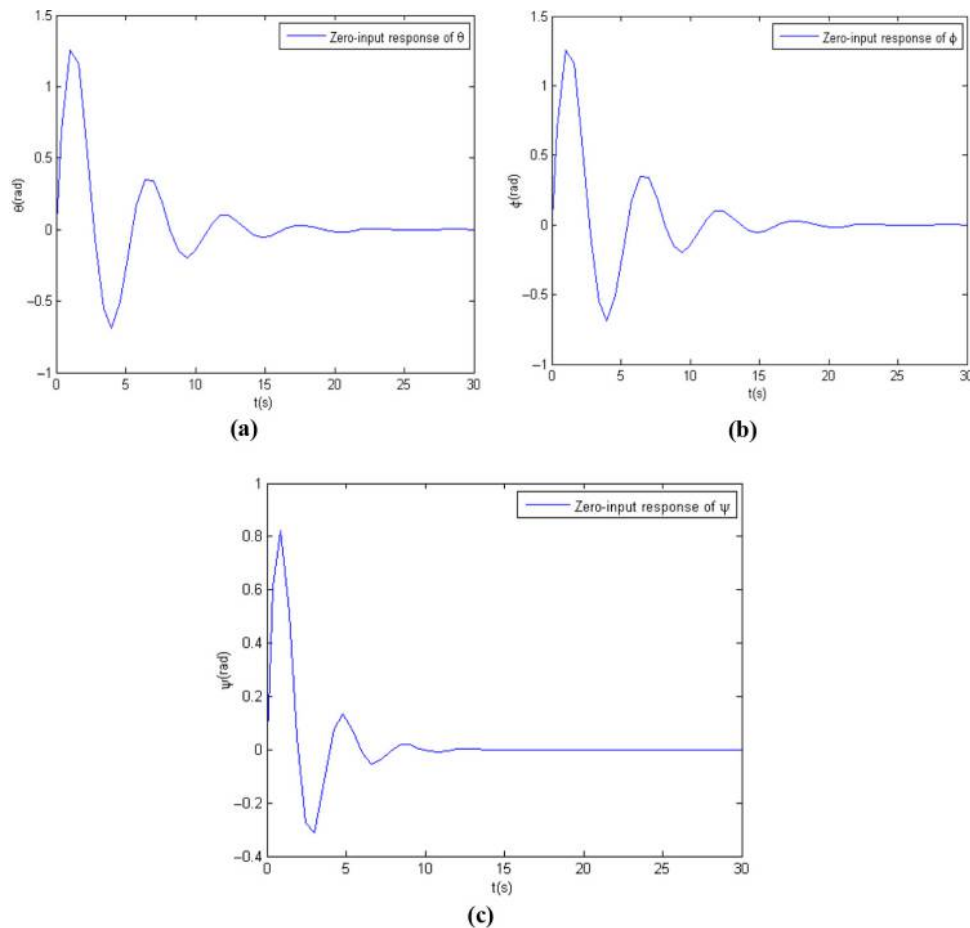
Map and compass operator

When the evolutionary iteration is less than the map and compass maximum iteration, the algorithm relies on the map and compass operator (Figure 5), which means the pigeons are far from the destination. Each pigeon has a position and a velocity of evolution. Suppose the position and the velocity of pigeon i are X_i, V_i . For a n -dimension search space, $X_i = [x_i^1, x_i^2, \dots, x_i^n]$, $V_i = [v_i^1, v_i^2, \dots, v_i^n]$. X_i and V_i are updated in every iteration. The new position X_i and velocity V_i of pigeon i at the t -th iteration are updated as follows:

$$V_i(t) = V_i(t-1) \cdot e^{-Rt} + rand \cdot (X_g - X_i(t-1)) \quad (16)$$

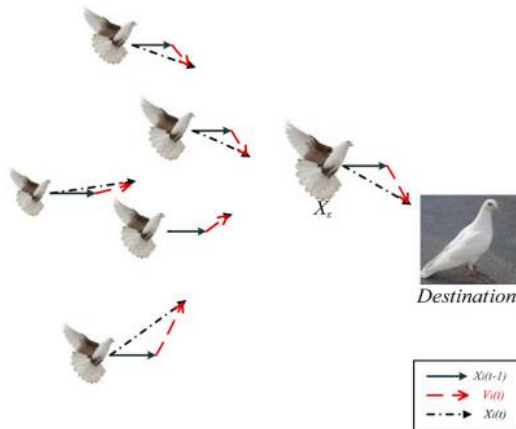
$$X_i(t) = X_i(t-1) + V_i(t-1) \quad (17)$$

Figure 4 The zero-input responses of attitudes



Notes: (a) The responses of pitch angle; (b) the responses of roll angle; (c) the responses of yaw angle

Figure 5 The process of map and compass operator evolution



where R is the map and compass factor which makes the velocity of evolution slow down as the iteration goes. $rand$ is a rand number within $[0, 1]$. X_g is the global best position, which means the maximum fitness value among all the pigeons.

Landmark operator

During the procedure of landmark operator, pigeons would fly straight to their destination if they are familiar with the landmarks. However, suppose the pigeons are still far from the destination, they are unfamiliar with the landmarks. The pigeons far from the destination (pigeons outside the big circle in Figure 6) would follow those that are familiar with the landmarks. During landmark operator, half of the pigeons would regard the center of the pigeons as their destination, and they would fly straight to the center, as the pigeons in the big circle in Figure 6. Thus, the number of pigeons would be decreased a half in every iteration. Let $X_c(t)$ be the center of some pigeons at the t -th iteration. The position of pigeon i at the t -th iteration can be calculated by the following equation:

$$N_p(t) = \frac{N_p(t-1)}{2} \tag{18}$$

Figure 6 The process of landmark operator evolution



$$X_c(t) = \frac{\sum_{N_p} X_i(t) \cdot fitness(X_i(t))}{\sum fitness(X_i(t))} \tag{19}$$

$$X_i(t) = X_i(t-1) + rand \cdot (X(t) - X_i(t-1)) \tag{20}$$

where $fitness()$ is the fitness function, which can determine the quality of each pigeon.

The implementation procedure of PIO is presented as follows:

- *Step 1. Initialization of the pigeons and parameters:* Initialize pigeon's positions, velocities and the parameters of this algorithm as Table II.

Figure 7 The flowchart of basic PIO algorithm

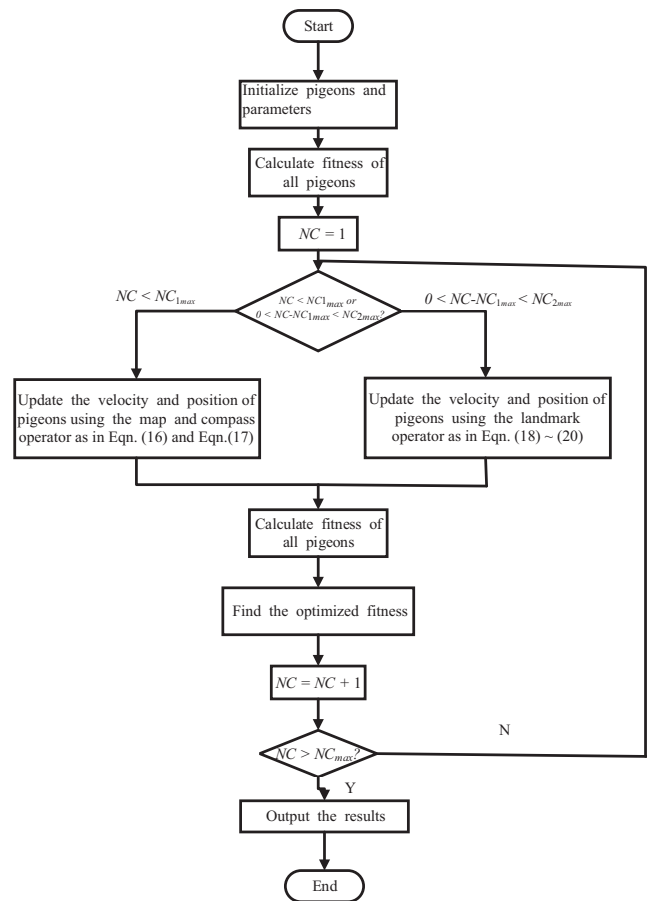


Figure 8 Structure of pose control system

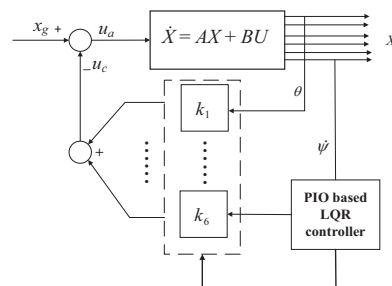


Figure 9 The implementation procedure of our proposed approach

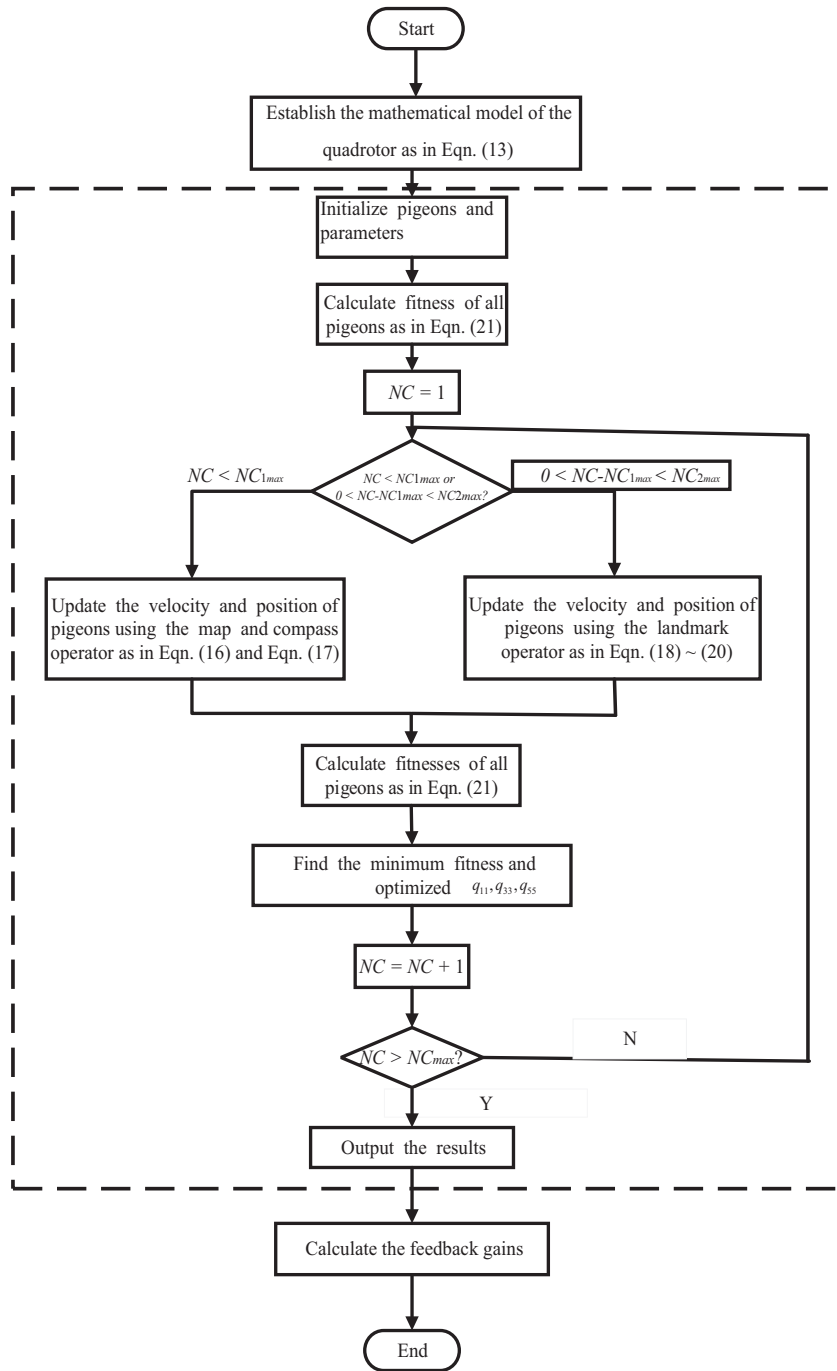


Table II The parameters of PIO algorithm

Parameter	Description	Value
n	Number of pigeons	200
NC_{max}	Maximum times of iteration	150
$NC1_{max}$	The iteration of map and compass operator	100
$NC2_{max}$	The iteration of landmark operator	50
R	The map and compass operator	0.2
D	Dimension of the search problem	2

Table III The parameters of PSO algorithm

Parameter	Description	Value
n	Number of particles	200
NC_{max}	Maximum times of iteration	150
c_1/c_2	Learning coefficient	1.8/1.3
k	Constriction factor	0.45
D	Dimension of the search problem	2

Figure 10 The evolution curves of PIO and PSO

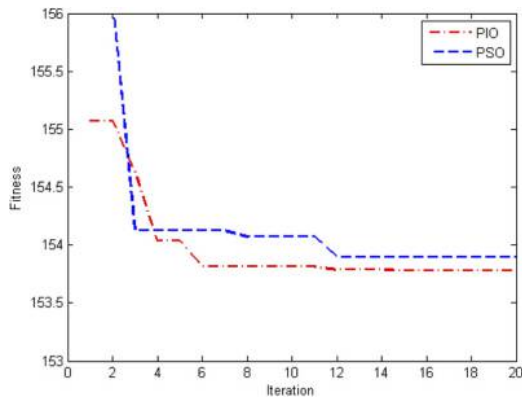
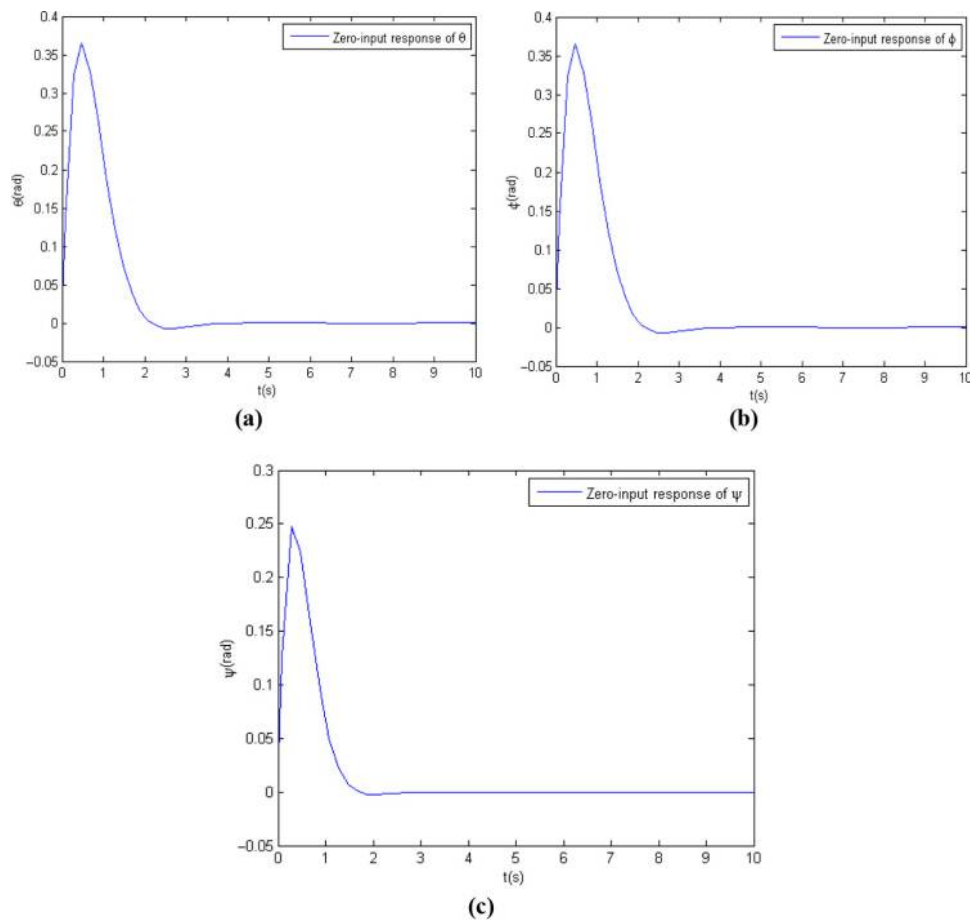


Table IV The parameters of the PIO based LQR approach

Parameter	Optimized value
Q	$\text{diag}(8.4997, 0, 8.4997, 0, 4.5395, 0)$
K	$[8.1829, 12.3224, 8.1829, 12.3224, 4.0623, 8.3069]$

Figure 11 The zero-input responses of pose angle



Notes: (a) The responses of pitch angle; (b) the responses of roll angle; (c) the responses of yaw angle

- Step 2. Calculate each pigeon's fitness value.
- Step 3. Update the pigeons: When $NC \leq NC1_{\max}$, update the pigeons using the map and compass operator. The velocity and position of each pigeon by equations (16) and (17). When $0 < NC - NC1_{\max} \leq NC2_{\max}$, update the pigeons using the landmark operator. The velocity and position of each pigeon updated by equations (18), (19) and (20).
- Step 4. Calculate each pigeon's fitness value: find out the optimized fitness.
- Step 5. Terminate whether the current number of iterations NC reaches the NC_{\max} , output the results: Otherwise, go to Step 3 (Figure 7).

Proposed controller

The key problem of LQR control is to select an appropriate control vector $u(t)$ so that the given quadratic performance index, equation (21), obtains the minimum value. It has been proved that the quadratic performance index presented in equation (21) shall reach the minimum value by means of linear control law in equation (22):

$$\mathcal{J} = \int_0^{\infty} (X^T Q X + u^T R u) dt \quad (21)$$

$$u(t) = -KX(t) = R^{-1}B^T P X(t) \quad (22)$$

The optimal matrix P can be calculate from algebraic Riccati equation (23):

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (23)$$

Matrices Q and R are chosen to have the form of:

$$Q = \text{diag}(q_{11}, 0, q_{33}, 0, q_{55}, 0), R = \text{diag}(1, 1, 1, 1, 1, 1) \quad (24)$$

where q_{11} , q_{33} and q_{55} are to be optimized by the given algorithm PIO. If one element of Q increases, the rapid responses of the system are improved. However, the oscillation of system would intensify and the energy cost of system would increase. If one element of R increases, the amplitudes of controlled variables would decrease and the energy cost would decrease. But the changes of dynamic

performances are tiny. In this paper, R is constant. The tuning parameters Q are optimized for the better performances.

$$u = -KX = R^{-1}B^T P X(t) = -(k_1 x_1 + \dots + k_6 x_6) \quad (25)$$

where the control input u is calculated from a full state feedback, of which the feedback gains $[k_1, k_2, k_3, k_4, k_5, k_6]$ are calculated from the optimized LQR.

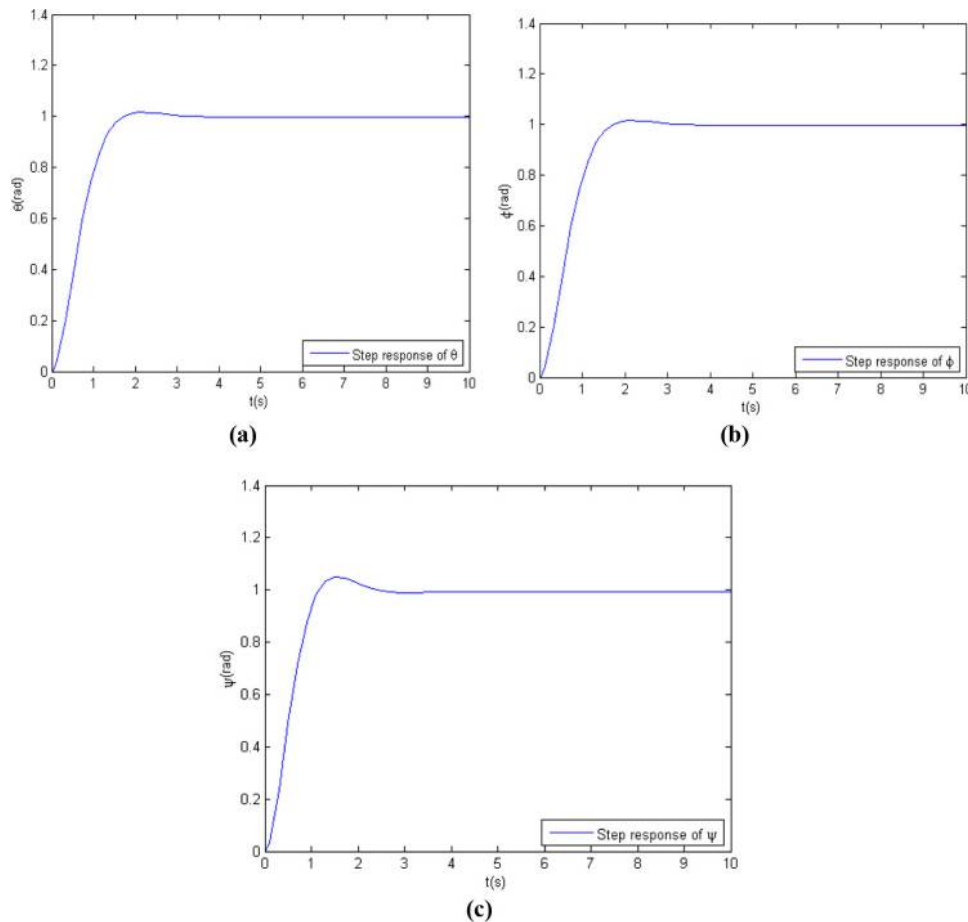
The PIO-optimized control system based on LQR is presented as Figure 8.

where u_c is the control input, which combine all the states $x_1, x_2, x_3, x_4, x_5, x_6$; $k_1, k_2, k_3, k_4, k_5, k_6$ are the feedback gains derived from the LQR approach; x_g can be seen as a given factor; u_c is the actual control signal acting upon the system.

The implementation procedure of our proposed PIO optimized LQR controller for attitude can be described as follows:

- Step 1. Establish the mathematical model of the quadrotor.
- Step 2. Set the performance index \mathcal{J} , the weighting matrices Q and R .
- Step 3. Optimize the designed control law using the PIO algorithm (Figure 7), where the parameters q_{11} , q_{33} and q_{55} are optimized using the fitness function:

Figure 12 The responses of attitude with a constant given



Notes: (a) The responses of pitch angle; (b) the responses of roll angle; (c) the responses of yaw angle

$$f := \int_0^{\infty} (X^T Q X + u^T R u) dt$$

- Step 4. Solve the algebraic Riccati equation (23) to get the matrix P .
- Step 5. Calculate the feedback gain vector:

$$K = -R^{-1}B^T P$$

- Step 6. Obtain the optimized control law:

$$u(t) = -KX(t) = R^{-1}B^T P X(t)$$

The flow chart of the proposed control law is as shown in Figure 9.

Simulation results

Due to the external disturbances, the system gets an initial state $x = [0, 2, 0, 2, 0, 2]$, which can lead to the unstable responses with sub-optimal parameters. The units of the state vector x are *rad/s*.

Tables II and III present the simulation parameters of PIO and PSO. The evolution curves of PIO and PSO are shown in Figure 10. The red and blue curves, respectively, represent PIO and PSO. As we can see that the fitness values decrease as the generation iterates with time, and the less fitness values stand for the better optimal parameters. The PIO algorithm can find out the optimal results faster than PSO and the fitness value of PIO is less than that of PSO, which demonstrates the effectiveness of PIO. The cost function is as equation (21). The less fitness value the cost function gets, the better performance the optimized controller has.

The optimal weight matrix Q and the resulting feedback gains K are shown in Table IV, which can produce best zero-input responses as Figure 11. In Figure 11, the amplitudes of pose angles responses decrease from about 1 to 0.35, and the steady time decreases from more than 10 s to about 3 s. The optimal parameters improve the performance of zero-input responses.

Furthermore, we apply a constant given to the controller. In Figure 12, we can see that the responses of three pose angle can stabilize at about 3 s. And the overshoot of responses is quite small. In this case, the quadrotor can respond quickly and steadily. Therefore, the quadrotor can be used to AAR, flight formation and detection.

Conclusions

Attitude control is a key procedure of flight control system. Therefore, an optimized control law is needed to realize the fast, efficient and steady responses. In this paper, our proposed PIO-based LQR controller for pose angles has good performances. The PIO algorithm can converge faster and obtain optimized fitness value. Comparison with PSO is conducted to verify the efficiency of PIO. The PIO algorithm can find out the optimized parameters faster and better than the PSO algorithm in LQR optimization. Thus, our proposed PIO-based LQR control for attitudes is an appropriate approach.

Future work

Our future work will focus on how to apply the control law and the PIO algorithm to the actual AAR procedure, which can save the time of debugging parameters. Moreover, the flight formulation and detection can adopt the control law as well.

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Further reading

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