A Multi-objective Pigeon-Inspired Optimization Algorithm Based on Decomposition

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Abstract. Multi-objective evolutionary algorithms based on decomposition (MOEA/Ds) convert a multi-objective optimization problem (MOP) into a set of scalar sub-problems, which are then optimized collaboratively. This paper designs a multi-objective pigeon-inspired optimization algorithm based on decomposition (MPIO/D) to improve the search efficiency by using the mechanism behind the remarkable navigation capacity of homing pigeons. The map and compass operator can record the direction of descent (rising) to generate good offspring. The landmark operator is used to accelerate the convergence of sub-problems with poor convergence. Compared to six competitive MOEA/Ds, MPIO/D has shown the advantages in tacking two benchmark sets of MOPs.

Keywords: Multi-objective evolutionary · Decomposition · Pigeon-inspired optimization

1 Introduction

In real-world applications, many problems need to simultaneously optimize multiple objectives that are typically characterized by conflicting objectives. These problems are called multi-objective optimization problems (MOPs). A minimized MOP which often has two or three objectives can be defined as follows [1]:

$$\begin{cases} \min F(x) = (f_1(x), f_2(x), \cdots, f_m(x)) \\ \quad\quad\quad \text{s.t.} \quad x \varepsilon \Omega \end{cases} \tag{1}$$

Where $\Omega \subseteq R^n$ is a n-dimensional decision space; $x = (x_1, \cdots, x_n) \in \Omega$ is a n-dimensional decision variable; $F : \Omega \to R^m (m = 2\, or\, 3)$ contains m inter-conflicting objective functions. The image of all Pareto optimal solutions is known as the Pareto front (PF). However, due to the conflicting nature of multiple objectives, there is no algorithm to obtain a single optimal solution that can optimize all objectives. Instead, some solutions can be obtained as a trade-off between different objectives, called the Pareto set (PS). The PS is termed the Pareto front (PF) in the objective space. To approximate the Pareto optimal set, great quantities of multi-objective evolutionary algorithms (MOEAs) [2, 3] have been proposed to solve MOPs in the past several decades. These algorithms can be roughly classified into three categories.

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The first category is the dominance-based MOEAs, which keep the non-dominated solutions and remove the dominated solutions and crowd non-dominated in the population [4]. These algorithms use Pareto dominance or modified Pareto dominance to distinguish and select candidate solutions. For example, the elitist non-dominated sorting genetic algorithm (NSGA-II) [5], the knee point has driven evolutionary algorithm (KnEA) [6], the Pareto envelop-based selection algorithm II (PESAII) [7], the strength Pareto evolutionary algorithm 2 (SPEA2) [8], RPD-NSGA-II [9] are representative MOEAs of this type. In these algorithms, all non-dominated solutions are firstly identified, and then the other strategy is used to make selections among the non-dominated solutions to maintain the population diversity.

The second category uses a decomposition-based method to solve MOPs. The main idea is to decompose a many-objective optimization problem into a set of sub-problems and optimize them collaboratively. The most representative algorithms are MOEA/D [10] and its variants [11–13]. And there are also some other methods based on decomposition, such as MOEA/D-M2M [14], DBEA [15], and so on [16–18]. These approaches are adept in diversity maintenance and avoiding local optimum but ineffective in addressing highly irregular PFs.

The third approach is indicator-based evolutionary algorithms. Indicators such as hypervolume [19] weigh both convergence and diversity of solutions to enhance the selection pressure and guide PF search. IBEA [20], SMS-EMOA [21], and HypE [22] are three classical indicator-based evolutionary algorithms. Unfortunately, the computational cost becomes excessively expensive because of the high computational complexity.

In 2014, Duan and Qiao proposed a novel bio-inspired computing algorithm named Pigeon-Inspired Optimization (PIO) [23]. Since the PIO algorithm in 2014, it had attracted much attention in the evolutionary algorithms research community. Many works [24, 25] develop and apply the PIO algorithm. Some multi-objective PIO algorithms [26, 27] are proposed to solve MOPs. In these multi-objective PIO algorithms, the population is updated by new solutions after solutions are clustered, which may low the speed of convergence.

This paper designs a multi-objective pigeon-inspired optimization algorithm based on decomposition (MPIO/D) to solve MOPs. The mechanism behind the remarkable navigation capacity of homing pigeons is used to improve the search efficiency. The map and compass operator can record the direction of descent (rising) to generate good offspring. The landmark operator is used to accelerate the convergence of sub-problems with poor convergence.

2 Basic Pigeon-Inspired Optimization

Pigeon-inspired optimization [23] uses two operators to imitate the behavior of homing pigeons, map and compass operator and landmark operator, respectively. Map and compass operators will be introduced as follows. Pigeons are randomly initialized with a D-dimension search space. The positions and velocities of pigeons respectively are denoted as $X_i = [x_{i1}, \cdots, x_{iD}]$ and $V_i = [v_{i1}, \cdots, v_{iD}]$, where $i = 1, \cdots, N$, N is the number of pigeons. The new positions X_i and V_i are updated as follows:

$$V_i^{t+1} = V_i^{t+1} * e^{-R*t} + rand * \left(X_{gbest} - X_i^t\right)$$

$$X_i^{t+1} = X_i^t + V_i^t \tag{2}$$

where R is the map and compass factor which is set to be between 0 and 1, t is the current times of iterations, X_{gbest} is the global best position that is located by comparing the positions of all the pigeons.

The landmark operator will have a total number of pigeons N in every generation. The pigeons in the lower half of the line sorted by fitness values are abandoned. The center of pigeons' position X_{center} is regard as the destination that every pigeon will fly to. The position X_i are generated according to Eq. (3):

$$X_{center}^t = \frac{\sum_{i=1}^{K} X_i^t * F\left(X_i^t\right)}{\sum_{i=1}^{K} F\left(X_i^t\right)}$$

$$\mathrm{K} = \mathrm{K}/2$$

$$X_i^{t+1} = X_i^t + rand * \left(X_{center}^t - X_i^t\right) \tag{3}$$

where $F\left(X_i^t\right)$ is the fitness value of X_i^t. After t_{max} iterations are completed, the landmark operator will stop.

3 The Proposed Algorithm

This paper combines the decomposition technology and pigeon-inspired optimization algorithm (MPIO/D) to address MOPs. This paper's main motivation is to design a MOEA to achieve a set of solutions that evenly distribute on the true PF. PIO algorithm uses the center of pigeons' position to address the problems, which can help MOEAs to improve their performance. MOEAs use neighboring information to the optimal population, which can improve the performance of MOEAs. In this paper, this optimization idea is utilized to enhance the performance of multi-objective PIO algorithm. First, new solutions update the population by using the updated strategy of MOEA/D [10]. Then, each sub-problem uses its neighboring solution as the global best position to optimize the neighboring sub-problems. Second, the neighboring solutions are used to update the center of pigeons' position of each sup-problem.

The MOEA/D decomposes a MOP into a series of sub-problems by weight vectors and aggregate functions. For each solution, it and some of its neighbor solutions are selected as parents to generate a new solution. Then, some of its neighbor solutions are updated by aggregate functions and the new solution. So, all sub-problems are simultaneously optimized in a population evolution. Each aggregate function makes some solutions converge to the corresponding weight vector, improving the convergence of the algorithm. Besides, the diversity of solutions is maintained by the uniformly distributed weight vectors. The main advantages of MOEA/D are that the given weight vectors can determine the diversity obtained; the neighbor's information is used to generate new offspring, improving the search efficiency.

The pseudo-code for the multi-objective pigeon-inspired optimization algorithm based on decomposition (MPIO/D) is displayed as follows:

The pseudo-code of the algorithm MPIO/D

Input :

MOP (1)

A stopping criterion

N: the number of direction vectors

K: the number of pigeons to determine the center of pigeons' position

T: the number of the neighborhood, $0 < T < N$

$\lambda^1, \lambda^2, \cdots, \lambda^N$: a set of N uniformly distributed weight vectors

Output: Objective vectors: $\{F(x^K), F(x^{2*K}), \cdots, F(x^{N*K})\}$

Initialization: Generate an initial population $x^1, x^2, \cdots x^N$ and an initial set of velocities $V^1, V^2, \cdots V^N$ with size N; determine $Z = (z_1, \cdots, z_m)$; randomly cluster the initial population into N clusters with size K and determine the best solution of each cluster; determine $B(i) = \{i_1, \cdots, i_N\}, (i = 1, \cdots, N)$, where $\lambda^{i_1}, \cdots, \lambda^{i_N}$ are the T closest weight vectors to λ^i, λ^{i_j} is closer than $\lambda^{i_{j+1}}$ to λ^i; $t = 1$.

While *the function evaluation times are less than the maximum function evaluation times* **do**

  For $i = 1, \cdots, N$, **do**

    Randomly select one solution from $\{x^{\lambda^{i_1}}, \cdots x^{\lambda^{i_T}}\}$ as the X_{gbest} to generate V_i^{t+1} and X_i^{t+1} by Eq. (2).

    Update of Z: For $j = 1, \cdots, m$, if $z_j < f_j(X_i^{t+1})$, then set $z_j = f_j(X_i^{t+1})$

    Update of Neighboring Solutions: set $H = 0$ and $Q = \{i_1, \cdots, i_T\}$

    While $H \le 2$ and $Q \ne \emptyset$

      $k = Q(1)$;

      If $g^{TE}(X_i^{t+1} | \lambda^k, z) < g^{TE}(x^k | \lambda^k, z)$

      set $x^k = X_i^{t+1}$, $V_k^{t+1} = V_i^{t+1}$ and $F(x^k) = F(X_i^{t+1})$. $H = H + 1$, .

      End if

      $Q(1) = []$;

    End while

      $t = t + 1$

    Uses $x^{\lambda^{i_1}}, \cdots x^{\lambda^{i_K}}$ to generate the position X_i accord to Eq. (3).

    Update of Z: For $j = 1, \cdots, m$, if $z_j < f_j(X_i)$, then set $z_j = f_j(X_i)$

      If $g^{TE}(X_i | \lambda^i, z) < g^{TE}(x^i | \lambda^i, z)$

      set $x^i = X_i$ and $F(x^i) = F(X_i)$. $H = H + 1$.

      End if

      If mod(t, 50)==0

        set $K = K/2$

      End if

  End for

end while

In this MPIO/D, the MOPs are solving by updating the neighboring solutions of each solution. We use the aggregation function $g^{TE}\left(x|\lambda^i, Z^*\right) = \max_{1\le j\le m}\left\{\left|f_j(x) - z_j^*\right|/\lambda_j^i\right\}$ to update the neighboring solutions of each solution, so the population is being updated simultaneously. This updated strategy makes solution converge to sub-problems $g^{TE}\left(x|\lambda^i, Z^*\right)$, this can ensure the convergence of the algorithm.

4 Experimental Studies

In this section, the performance of MPIO/D will be verified by comparing it with existing multi-objective optimization algorithms, e.g., NSGAII [5], MOEA/D [10]. Seven DTLZ problems [35] are used in this experiment. The number of decision variables is placed to 7, 21, 12, 22 for DTLZ1 and DTLZ3, DTLZ2 and DTLZ4-DTLZ6, DTLZ7, respectively. The inverted generational distance (IGD) [36] is used as a performance metric to quantify algorithms' performances. Moreover, the Wilcoxon Rank-Sum test [38] is employed at a significance level of 0.05. It tests whether the performance of MOIO/D is significantly better ("+"), statistically similar ("="), or significantly worse ("−") than/as that obtained by the compared algorithms.

All algorithms are implemented by using the MATLAB language and run independently for thirty times with the maximal number of function evaluations 100 000 on all test problems. For fair comparisons, the population size and the maximal number of function evaluations of the compared algorithms are the same as this work, and other parameters of NSGAII and MOEA/D are the same as the original literature. In MPIO/D, $R = 5$ and $K = 0.9N$; the population size is set to 105 for all compared algorithms on each test problem; the size of neighborhood list T is set to 0.1N, the probability of choosing mate sub-problem from its neighborhood J is set to 0.9.

This subsection presents the comparison results on IGD in seven problems. Table 1 gives the mean and standard deviation values for three comparison algorithms. We can obtained from the Table 1 that, in the form of the IGD metric, the results obtained by MPIO/D are better than those obtained by NSGAII and MOEA/D on more than five problems, which indicates that the final solutions obtained by MPIO/D have a better diversity than those obtained by NSGAII and MOEA/D, and have a good convergence. Moreover, MPIO/D outperforms NSGAII and MOEA/D in solving DTLZ1 and DTLZ3. This emphasizes that the selection strategy and crossover operators have the advantage in solving multiple local fronts problems. NSGAII is better at solving DTLZ5. The higher mean IGD value for MPIO/D is because the update strategy is not suitable for MOPs with degenerated PF.

"+" means that MPIO/D outperforms its competitor algorithm, "−" means that MPIO/D is worse than its competitor algorithm, and "=" means that the competitor algorithm has the same performance as MPIO/D.

Table 1. IGD, GD and HV obtained by MPIO/D, MOEA/D and NSGAII

Problems		IGD	
		Mean	Std
DTLZ1	MPIO/D	**0.0188**	0.0001
	MOEA/D	0.0314(+)	0.0016
	NSGAII	0.0356(+)	0.0500
DTLZ2	MPIO/D	**0.0584**	0.0035
	MOEA/D	0.0813(+)	0.0053
	NSGAII	0.0692(+)	0.0021
DTLZ3	MPIO/D	**0.0645**	0.0515
	MOEA/D	0.0807(+)	0.0048
	NSGAII	0.0692(+)	0.0024
DTLZ4	MPIO/D	**0.0561**	0.0031
	MOEA/D	0.0822(+)	0.0053
	NSGAII	0.1299(+)	0.1628
DTLZ5	MPIO/D	0.0114	0.0015
	MOEA/D	0.0121(+)	0.0030
	NSGAII	**0.0053**(−)	0.0003
DTLZ6	MPIO/D	0.0197	0.0003
	MOEA/D	**0.0118**(−)	0.0038
	NSGAII	0.0555(+)	0.0260
DTLZ7	MPIO/D	**0.0853**	0.0007
	MOEA/D	0.1558(+)	0.0239
	NSGAII	0.1124(+)	0.0935

5 Conclusions

In this paper, we proposed a multi-objective pigeon-inspired optimization algorithm, called MPIO/D, based on the idea of decomposition. In this approach, MOEA/D [10] update strategy is used in the MPIO, balancing diversity and convergence. Moreover, MPIO/D comperes with NSGAII, MOEA/D on some test sets with complicated PS or many local PFs. According to the performance analyses, MPIO/D shows competitive performances on most test problems against for comparisons MOEAs. However, for a few benchmark functions, the proposed algorithm shows shortcomings because the update strategy is not suitable for some MOPs with degenerated PF. In the future, we will study that this algorithm is used to solve real-world problems.

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