# Quadrotor Flight Control Parameters Optimization Based on Chaotic Estimation of Distribution Algorithm

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**Abstract.** Quadrotor is a type of rotor craft that consists of four rotors and two pairs of counter-rotating, fixed-pitch blades located at the four corners of the body. The flight control parameters optimization is one of the key issues for quadrotor. Estimation of distribution algorithm is a new kind of evolutionary algorithm developed rapidly recently. However, low convergence speed and local optimum of the EDA are the main disadvantages that limit its further application. To overcome the disadvantages of EDA, a chaotic estimation of distribution algorithm is proposed in this paper. It is a combination of chaos theory and principles of estimation of distribution algorithm. Series of experimental comparison results are presented to show the feasibility, effectiveness and robustness of our proposed method. The results show that the proposed chaotic EDA can effectively improve both the global searching ability and the speed of convergence.

**Keywords:** quadrotor, estimation of distribution algorithm, chaos, flight control.

## 1    Introduction

Quadrotor is a type of rotorcraft that consists of four rotors and two pairs of counter-rotating, fixed-pitch blades located at the four corners of the body. The idea of using four rotors is realized as a full-scale helicopter as early as 1920s [1]. However, quadrotor is dynamically unstable and not widely developed in applications until the advance in computers and micro sensors. Flight control system of quadrotor is a complex MIMO nonlinear system with time-varying, strong-coupling characteristics[2].Though we can rely on small disturbance linearization equation to design the control system, the apparent coupling among the equations will make it difficult to set the parameters.

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Estimation of distribution algorithm is novel kind of optimization algorithm. It is a combination of genetic algorithms and statistical learning [3]. Nowadays it has become a significant method dealing with programming problems such as the optimization of flight control system. Besides, estimation of distribution algorithms was put forward as a significant issue in almost every academic seminar such as ACMEVO, IEEE and CEC. Nevertheless, it can easily trap into the local optimum, hence would probably end up without finding a satisfying result. Considering the outstanding performance of chaos theory in jumping out of stagnation, we introduce it to improve the robustness of basic EDA algorithm, and the comparative experimental results testified that our proposed method manifests better performance than the basic EDA algorithm. We also applied the chaotic EDA to flight parameters optimization of quadrotor, whose type is AR.Drone.

The remainder of this paper is organized as follows. Section 2 introduces the modeling of the quadrotor. Subsequently, section 3 describes the chaotic estimation of distribution algorithm. Experimental results are given in section 4. Our concluding remarks are contained in section 5.Acknowledgements are contained at the end.

## 2      Modeling of the Quadrotor

AR.Drone is a Wi-Fi-controlled quadrotor with cameras attached to it which is developed by Parrot Inc [1]. It uses an ARM9 468MHz embedded microcontroller with 128M of RAM running the Linux operating system [4].The onboard downward Complementary Metal Oxide Semiconductor (CMOS) color camera provides RGB images in size of 320*240. An inertial system uses a 3-axisaccelerometer, 2-axis gyro and a single-axis yaw precision gyro. An ultrasonic altimeter with a range of 6 meters provides vertical stabilization. With a weight of 380g or 420git can maintain flight for about 12 minutes with a speed of 5m/s. Fig.1 shows the top view and side view of the quadrotor.



**Fig. 1.** Top view and side view of the quadrotor

1. Transform from angle to voltage[5]

$$u(t) = K_1\theta(t) \tag{1}$$

2. Transform from voltage to torque

The relationship between voltage and torque is:

$$\begin{cases} T_m \dfrac{dw(t)}{dt} + w(t) = K_m u(t) \\ M_m = K_2 \dfrac{u}{R} \\ M_f = fw(t) \\ M = M_m - M_f \end{cases} \tag{2}$$

3. Transform from torque to pneumatic tension

$$M = Fr \tag{3}$$

4. Transform from pneumatic tension to the tilt angle of the quadrotor

$$\begin{cases} M = J_m \dfrac{d^2\theta}{dt} \\ M = 2Xl \\ J_m = \dfrac{1}{3} m_1 l^2 + 2m_2 l^2 \end{cases} \tag{4}$$

From 1, 2, 3, 4 we can get the following result through Laplace transformation.

$$G(s) = G_1 G_2 G_3 G_4 = \frac{6226.s + 311330}{s^3 + 100s^2} \tag{5}$$

The PID controller architecture is shown in Fig. 2 [1].

The input of the controller is the errors, which can be obtained by our proposed chaotic EDA. The output of the controller is:

$$u = k_p e(t) + k_i \int_0^\infty e(t)dt + k_d \frac{de(t)}{dt} \tag{6}$$

Considering there is no apparent relationship between the inputs and the outputs of the flight control system and in order to avoid the overshoot, we choose the following objective function [12].

$$\begin{cases} J = \int_0^\infty (w_1 \, | \, e(t) \, | + w_2 u^2(t))dt + w_3 t_u, e(t) >= 0 \\ J = \int_0^\infty (w_1 \, | \, e(t) \, | + w_2 u^2(t) + w_4 \, | \, e(t) \, |)dt + w_3 t_u, e(t) < 0 \end{cases} \tag{7}$$

Where $J$ is the objective function, $e(t)$ represents the error, $u(t)$ denotes the outputs of the controller, $t_u$ means the rising time and $w_1$, $w_2$,$w_3$ and $w_4$ are the weights and $w_4 \gg w_1$.
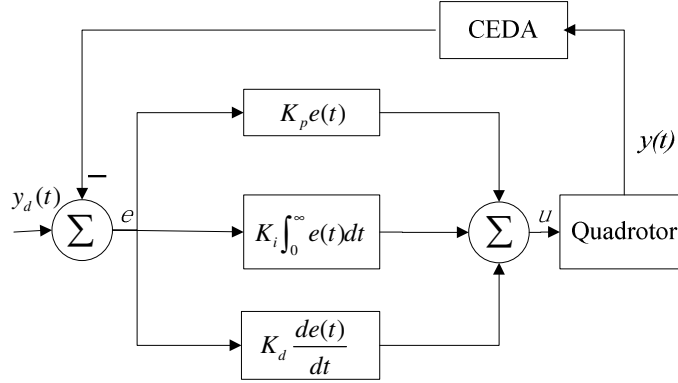


**Fig. 2.** The PID controller architecture

# 3      Chaotic Estimation of Distribution Algorithm(CEDA)

### 3.1      Principles of CEDA

The term of EDA alludes to a family of evolutionary algorithms which represents an alternative to the classical optimization methods in the area [6], [7], [8]. EDA generates new population by establishing probability distribution model and generate new individuals based on the model. Indeed, this distribution is responsible for one of the main characteristics of these algorithms. The basic procedures can be shown as follows.

Given an n-dimensional probability vector model $P(x) = P(x_1, x_2,…,x_n) = (0.5, 0.5,…,0.5)$. Then generate initial population based on the model. We conduct the selection operation and select m<n individuals to update the model by the following formula:

$$P_{l+1}(x_i) = \frac{P_l(x_i)}{\sum_{i=1}^{m} P_l(x_i)} \qquad (8)$$

Where $P$ means the probability, $l$ represents the evolutionary times and m denotes the better m individuals selected from the former population. Repeat the selection and updating operation until reaching the stopping criteria.

Chaos is the highly unstable motion of deterministic systems infinite phase space which often exists in nonlinear systems [9].Chaos theory is epitomized by the so-called 'butterfly effect' detailed by Lorenz [10]. Until now, chaotic behavior has already been observed in the laboratory in a variety of systems including electrical circuits, lasers, oscillating chemical reactions, fluid dynamics, as well as computer models of chaotic processes. Chaos theory has been applied to a number of fields, among which one of the most applications was in ecology, where dynamical systems have been used to show how population growth under density dependence can lead to chaotic dynamics. Sensitive dependence on initial conditions is not only observed in complex systems, but even in the simplest logistic equation. In the well-known logistic equation:

$$x_{n+1} = 4x_n(1 - x_n) \tag{9}$$

Where $0 < x_n < 1$, a very small difference in the initial value of $x$ would give rise to large difference in its long-time behavior, which is the basic characteristic of chaos. The track of chaotic variable can travel ergodically over the whole space of interest. The variation of the chaotic variable has a delicate inherent rule in spite of the fact that its variation looks like in disorder. Therefore, after each search round, we can conduct the chaotic search in the neighborhood of the current optimal parameters by listing a certain number of new generated parameters through chaotic process. In this way, we can make use of the ergodicity and irregularity of the chaotic variable to help the algorithm to jump out of the local optimum as well as finding the optimal parameters. The experimental results in section 4 show the efficiency of our algorithm.

### 3.2    CEDA Approach for Flight Control Parameters Optimization

Chaotic estimation of distribution algorithm (CEDA) is a combination of chaos theory and basic EDA. The CEDA is superior to the basic EDA mainly in the following aspects. The introduction of chaotic theory into basic EDA is an important improvement. EDA can converge fast, but sometimes the fast convergence happens in the first few iterations and relapses into a local optimum easily [11]. By introducing the chaos theory, we can avoid from the local optimum as well as to increase the speed of reaching the optimal solution. The detailed procedure of our proposed CEDA approach to the optimization of flight control parameters can be described as follows.

**Step1:** Initialize the detailed parameters of the estimation of distribution algorithms (EDA) such as the population size, coding length and so on.
**Step2:** Encode the variables in a proper way.
**Step3:** Initialize the probability distribution model $P_0(x)=P_0(x_1,x_2,\ldots,x_n)$ =(0.5, 0.5,…,0.5). ($l$=0). Then generate the initial population including $N$ individuals according to the $P_l(x)$.
**Step4:** Calculate the fitness of every individual according to formula (7) and select the best $M <= N$ individuals.
**Step5:** Conduct the chaotic search around the best solution based on formula (9). Among the engendered series of solutions, select the best one and use it to replace the former best solution.

**Step6:** Update the probability distribution model to $P_{l+1}(x)$ according to formula (8) and generate another population of new generation based on the model $P_{l+1}(x)$.

**Step7:** Echo the step4, step5 and step6 until reaching the stopping criteria.

**Step8:** Decode the variables and output the results.

## 4    Experimental Results

In order to investigate the feasibility and effectiveness of the chaotic estimation of distribution algorithm to the optimization of parameters, a series of comparative experiments have been conducted.

Our control object is the formula (5). The detailed parameters are set as follows. $w_1$=0.999, $w_2$=0.001, $w_3$=2.0, $w_4$=100. The population size is 30. The evolutionary times are 100. By means of Matlab, we can easily obtain the comparative results in Fig.3 and Fig.4.

It is noted that the'EDA' in Fig.3 and Fig.4 represents the simulation results of basic EDA while 'CEDA' denotes the results of chaotic EDA. It turns out that our method performs better than the basic EDA. As is shown in Fig.3, the objective function can converge to a smaller range with a faster speed by CEDA compared with basic EDA. While in Fig.4, we can see clearly that using CEDA can make the quadrotor to track the given signal faster and more steadily.

From the experimental results, it is obvious that our improved EDA can jump outof the local optimum as well as speeding up the process of finding the optimal parameters. The experimental results proves that our proposed method is a more feasible and effective approach in solving the problem of optimization of flight control parameters.
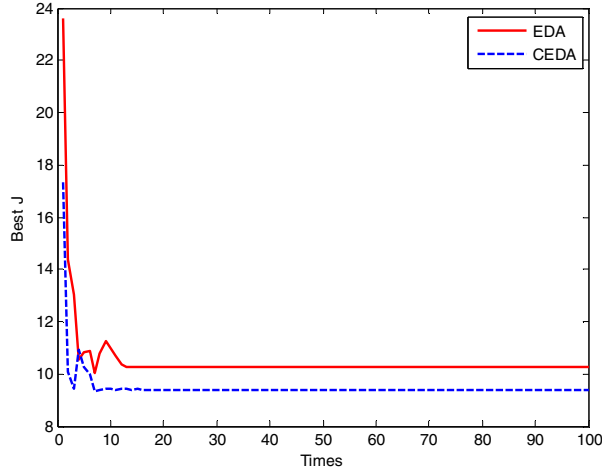


**Fig. 3.** Comparative objective function response curves by using EDA and CEDA
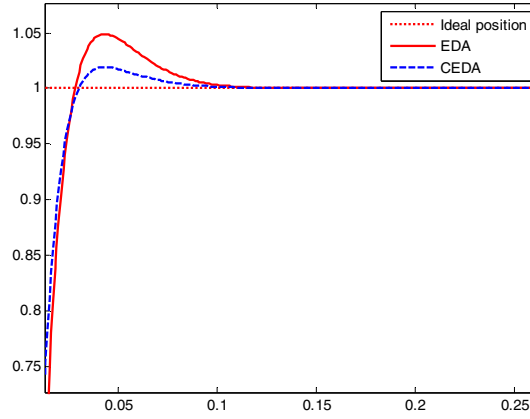
**Fig. 4.** Comparative results of step response curves by using EDA and CEDA

## 5    Conclusions

In this paper, an improved estimation of distribution algorithm-CEDA is proposed. The chaos theory is introduced into the basic EDA, and a better performance can be attained in this way. Comparative experimental results of the proposed CEDA and basic EDA are also given to verify the feasibility and effectiveness of our proposed approach, which provide a more effective way for the optimization of flight control parameters.

Our future work will focus on applying our proposed CEDA to the actual flight control system.

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