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An improved artificial physics approach to multiple UAVs/UGVs heterogeneous coordination

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This paper proposed an improved artificial physics (AP) method to solve the autonomous navigation problem for multiple unmanned aerial vehicles (UAVs)/unmanned ground vehicles (UGVs) heterogeneous coordination in the three-dimensional space. The basic AP method has a shortcoming of easily plunging into a local optimal solution, which can result in navigation fails. To avoid the local optimum, we improved the AP method with a random scheme. In the improved AP method, random forces are used to make heterogeneous multi-UAVs/UGVs escape from local optimum and achieve global optimum. Experimental results showed that the improved AP method can achieve smoother trajectories and smaller time consumption than the basic AP method and basic potential field method (PFM).

artificial physics, heterogeneous coordination, unmanned aerial vehicle (UAV), unmanned ground vehicle (UGV), autonomous navigation, obstacle avoidance

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1 Introduction

The artificial physics (AP) is widely used to design and build rapidly deployable, scalable, adaptive, cost-effective, and robust networks of autonomous distributed multi-agents due to its efficient mathematical analysis and simplicity [1]. The “artificial physics” force-based method for distributed control was firstly proposed by Spears et al. [2] inspired by the close connection between physics and distributed control. In the AP method approach, the control law for a robot navigating uses the attractive force and repulsive force to determine a vector that points toward the target, as has been identified sufficiently in refs. [2, 3].

Multiple unmanned aerial vehicles (UAVs)/unmanned ground vehicles (UGVs) heterogeneous coordination provides a new breakthrough for the effective application of UAV and UGV by using their strengths in full and complement each other perfectly [4]. For multiple UAVs/UGVs heterogeneous systems, navigation and dynamic path tracking is a typical problem. In order to realize the multiple UAVs and UGVs heterogeneous coordinated movement, the control law should be adapted to heterogeneous model [5], and multiple UGVs/UAVs should avoid collisions during movement [6]. A path-based navigation method was presented in two advanced research projects of UAV rotorcraft and UGV cooperating in the battle space currently approved for funding by Defence R&D Canada [7]. A cooperative control framework was proposed for a hierarchical UAV/UGV platform for wildfire detection and fighting [8]. In order to realize the multiple UAVs and UGVs heterogeneous coordinated movement, the navigation mechanism in the heterogeneous system should be further improved [9].

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The application of AP method, however, is often associated with the local minima problem when the total force acting on an agent is summed up to zero although the agent has not reached its goal position yet. Many researchers have investigated this problem, such as Gaussian transforming method [10] and developed the AP strategy [11]. However, these methods also cause a longer path. As the effects on the agent are calculated by assembling all the forces caused by the environment, a random force is introduced based on a random scheme, which can give the agent an opportunity to escape from the local optimum and achieve the global target finally.

This paper mainly proposes an improved artificial physics approach for multiple UGVs/UAVs navigation in a complicated three-dimensional environment. Complex navigation is achievable through simple local interactions by using this approach instead of traditional navigation methods. Since the approach is largely independent of the size and number of agents, the results can scale well to larger agents and larger sets of agents [12]. The feasibility and effectiveness of our proposed method are verified by comparative experiments with the basic AP and potential field method (PFM). Series of comparative results also show that it is an effective method for obstacle avoidance and navigation of the heterogeneous UAVs/UGVs.

The remainder of the paper is organized as follows. Section 2 introduces the basic principles of the general artificial physics. Section 3 gives a detailed description of the UAV/UGV model, and the AP based heterogeneous multiple UAVs/UGVs navigation scheme is also described in this section. Section 4 introduces the improved Newton algorithm and the random force scheme strategy. The comparative experimental results are given and analyzed in Section 5, and our concluding remarks are contained in the final section.

2 Artificial physics construction

In this section, the basic AP is formulated, which can be used to drive a UAV/UGV heterogeneous system to a desired configuration or state via the virtual physics forces [10]. The desired system state is a stable and equilibrium state that has a global minimum potential energy [11]. Consider the agent (UAV or UGV) as a point-mass in the three-dimensional space with a position vector and a velocity vector, which can be expressed as

$$\begin{aligned} \boldsymbol{p} &= (x, y, z)^{\mathrm{T}}, \\ \boldsymbol{v} &= (V_x, V_y, V_z). \end{aligned} \tag{1}$$

The movement of each agent is controlled based on the Newton's law:

$$\boldsymbol{F} = m\boldsymbol{a},$$

$$\begin{aligned} \Delta\boldsymbol{p} &= (\boldsymbol{v}_0 + \boldsymbol{v}_t)\Delta t/2, \\ \boldsymbol{v}_t &= \boldsymbol{v}_0 + \Delta\boldsymbol{v}, \\ \Delta\boldsymbol{v} &= \boldsymbol{F}\Delta t/m, \end{aligned} \tag{2}$$

where m represents the mass of each agent and $\boldsymbol{F}$ represents the virtual force. At each sampling time Δt, the movement of each agent depends on the average of the initial and terminal velocities, and the changes of these velocities are caused by the virtual physics forces.

In the AP framework, agents exert virtual forces upon others and respond to forces from other agents and the target. Imitating the Newtonian gravitational force law, the virtual force law between agents and the target can be defined as

$$\boldsymbol{F} = \frac{G}{\boldsymbol{r}^2}, \tag{3}$$

where $\|\boldsymbol{F}\| \leqslant F_{\max}$ represents the magnitude of the force, and $\boldsymbol{r}$ denotes the distance between agents or the distance between agents and the target. The "gravitational constant" G affects the strength of the force. The virtual force is repulsive if $r<R$ and attractive if $r>R$. Each agent has a sensor that detects the ranges and bearings to nearby agents. To ensure that the virtual forces are local in nature, the visual range of agents is limited to $1.5R$. The value of G, $F_{\max}$ and R must be determined as initial parameters.

Besides of the Newtonian force law, Hettiarachchi [13] investigated another force law, which can be regarded as a generalization of the Lennard-Jones (LJ) force law. The LJ force model between molecules and atoms can be defined as

$$\boldsymbol{F} = 24\varepsilon\left[\frac{2dR^{12}}{\boldsymbol{r}^{13}} - \frac{cR^6}{\boldsymbol{r}^7}\right], \tag{4}$$

where ε, d, c and R are parameters that must be determined in advance.

The Newtonian force law is generally used to create rigid formations that act as solids in a uncertain environment, while the LJ force law can easily model crystalline solid formations, liquids, and gases [14].

3 AP based heterogeneous multi-agents navigation method

In multiple UAVs/UGVs heterogeneous coordinated motion, UAVs and UGVs are all considered as particles. Therefore, these particles can be controlled based on the Newton's law mentioned above. Each particle in the heterogeneous system has its dynamic model (as a UGV or a UAV) with constraints and the velocities of the particles are generated based on the AP method. Assume the number of UGVs in the formation to be N and they are moving on a plane. The status variable of UGV$_i$ is represented as $\boldsymbol{x}_{\mathrm{UGV}_i} = (x_i, y_i, \dot{x}_i, \dot{y}_i)$, i=1, 2,⋯, N. Therefore, the formation system state can be defined as [4]

$$\begin{cases} \dot{\boldsymbol{p}}_i = \boldsymbol{v}_i, \\ \dot{\boldsymbol{v}}_i = \boldsymbol{u}_i. \end{cases} \tag{5}$$

Define $\boldsymbol{p}_i=(x_i, y_i)$ as the position vector of the ith UGV. The velocity vector and control input can be written as $\boldsymbol{v}_i = (\dot{x}_i, \dot{y}_i)$ and $\boldsymbol{u}_i=(u_{xi}, u_{yi})$.

In the heterogeneous navigation motion, the state vector of the jth UAV is $\boldsymbol{x}_{\mathrm{UAV}_j} = (v_j, \gamma_j, \chi_j, x_j, y_j, z_j)^{\mathrm{T}}$ for j=1, 2,⋯, N, and its control inputs are thrust $\boldsymbol{T}_j$, load factor $\boldsymbol{n}_j$ and bank angle $\boldsymbol{\mu}_j$. The equations of motion for the UAV are as follows:

$$\begin{aligned} &\dot{v} = g[(T-D)W - \sin\gamma], \\ &\dot{\gamma} = (g/v)(n\cos\mu - \cos\gamma), \\ &\dot{\chi} = (gn\sin\mu)/(v\cos\gamma), \\ &\dot{x} = v\cos\gamma\cos\chi, \\ &\dot{y} = v\cos\gamma\sin\chi, \\ &\dot{z} = -v\sin\gamma, \end{aligned} \tag{6}$$

where v is the air speed of UAV, γ is the flight path angle, χ the flight path heading, x, y, and z denote the position, g denotes the acceleration of gravity, D denotes the drag, and W denotes the weight.

Based on the AP framework, the motion behaviours of heterogeneous UAVs/UGVs can be classified into two fundamental types: obstacle-avoidance and goal-seeking [15]. The goal-seeking behaviour can be carried out by the virtual physics force law which attracts heterogonous agents to the goal. By using the Newtonian gravitational force law, the virtual attractive force on heterogonous UAVs/UGVs toward the goal can be defined as

$$\boldsymbol{F}_{\mathrm{att}} = \frac{G_{\mathrm{goal}}}{\left\|\boldsymbol{p}_{\mathrm{agent}} - \boldsymbol{p}_{\mathrm{goal}}\right\|^2} \hat{\boldsymbol{r}}_{\mathrm{agent}\to\mathrm{goal}}, \tag{7}$$

where $\left\|\boldsymbol{p}_{\mathrm{agent}} - \boldsymbol{p}_{\mathrm{goal}}\right\|$ represents the Euclidean distance between UAV/UGV and its goal, and $\hat{\boldsymbol{r}}_{\mathrm{agent}\to\mathrm{goal}}$ represents the unit vector from $\boldsymbol{p}_{\mathrm{agent}}$ to $\boldsymbol{p}_{\mathrm{goal}}$. G_{goal} is the gravitational constant of the goal.

The behavior of obstacle-avoidance is classified as the virtual repulsive force, which keeps heterogonous UAVs/UGVs away from obstacles. Considering the Newtonian gravitational force law, the repulsive force is defined as

$$\boldsymbol{F}_{\mathrm{rep}} = \begin{cases} \dfrac{G_{\mathrm{obs}}}{\left\|\boldsymbol{p}_{\mathrm{agent}} - \boldsymbol{p}_{\mathrm{obs}}\right\|^2} \hat{\boldsymbol{r}}_{\mathrm{obs}\to\mathrm{agent}}, & \left\|\boldsymbol{p}_{\mathrm{agent}} - \boldsymbol{p}_{\mathrm{obs}}\right\| \leqslant R_{\mathrm{rep}}, \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

where $\hat{\boldsymbol{r}}_{\mathrm{obs}\to\mathrm{agent}}$ represents the unit vector from the obstacle to heterogonous UAVs/UGVs. R_{rep} is the action range of the repulsive force. Therefore, the repulsive force would take action and push UAV or UGV away only when the heterogonous UAVs/UGVs move near enough to the obstacle [16].

Moreover, the rate limiter is introduced and the velocity of heterogonous UAVs/UGVs are limited to $\|\boldsymbol{v}\| \leqslant v_{\max}$. A frictional force can be expressed as

$$\boldsymbol{f} = -f\boldsymbol{v}. \tag{9}$$

Above all, the control law (or the navigation function) for the basic AP method that drives the heterogonous UAVs or UGVs to move consists of three virtual forces:

$$\boldsymbol{u} = \boldsymbol{F}_{\mathrm{att}} + \sum \boldsymbol{F}_{\mathrm{rep}} + \boldsymbol{f}, \tag{10}$$

where the mass of the agent is set as m=1. The virtual forces on the heterogonous UAVs/UGVs when moving through obstacle fields are shown in Figure 1.

4 Improved AP method

4.1 Improved Newton method

Actually, the AP method can be regarded as a gradient descent approach of the potential filed method. The AP forces are along with the negative gradient direction of the corresponding gravitational potential field. When heterogonous UAVs/UGVs are moving in a complex environment with multiple obstacles, the AP method can also result in serious oscillations. This phenomenon may cause an extremely long task completion time because of too many unnecessary control manoeuvres.

To solve this problem, the improved Newton method of the optimization theory is employed [17]. As in the optimization theory, Newton methods adopt the most important

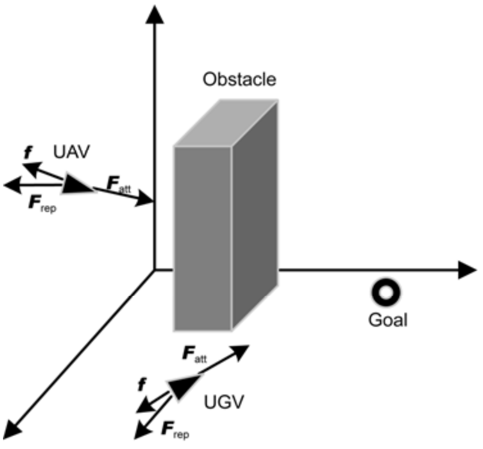

Figure 1 AP forces on heterogonous multiple UAVs/UGVs.

search direction−the Newton direction. This direction is derived from the second-order Taylor series approximation. Compared to the methods using the negative gradient as the search direction, Newton methods have a good convergence, fast solution speed and high precision. Motivated by this, we modify the basic AP method using Newton methods and change directions of AP forces toward the Newton directions.

The dynamics of heterogonous UAVs/UGVs based on the improved AP method is given by the control law:

$$\boldsymbol{u}=\boldsymbol{B}\left(\boldsymbol{p},\boldsymbol{F}_{\text{att}}\right)\cdot\boldsymbol{F}_{\text{att}}+\sum\boldsymbol{B}\left(\boldsymbol{p},\boldsymbol{F}_{\text{rep}}\right)\cdot\boldsymbol{F}_{\text{rep}}+\boldsymbol{f},\tag{11}$$

where matrix $\boldsymbol{B}\left(\boldsymbol{p},\boldsymbol{F}\right)$ is based on the improved Newton method. As in the optimization theory, $\boldsymbol{B}$ should be a positive-definite matrix and is defined as

$$\boldsymbol{B}=\left(\boldsymbol{G}+v\boldsymbol{I}\right)^{-1},\tag{12}$$

where the presence of v can guarantee the matrix invertible. $\boldsymbol{G}$ is the Hessian matrix of corresponding conservative force potential function, which can be defined as

$$\boldsymbol{G}=\begin{pmatrix}\dfrac{\partial F_x}{\partial x} & \dfrac{\partial F_x}{\partial y} & \dfrac{\partial F_x}{\partial z}\\ \dfrac{\partial F_y}{\partial x} & \dfrac{\partial F_y}{\partial y} & \dfrac{\partial F_y}{\partial z}\\ \dfrac{\partial F_z}{\partial x} & \dfrac{\partial F_z}{\partial y} & \dfrac{\partial F_z}{\partial z}\end{pmatrix},\tag{13}$$

where $\left(F_x,F_y,F_z\right)^{\mathrm{T}}$ is the components of force $\boldsymbol{F}$ along the x-axis, y-axis and z-axis. We can intuitively analyze the basic and the improved AP method from the optimization point of view. The basic AP method is essentially a kind of gradient descent method, which is inherently associated with the oscillation problem due to its orthogonal search directions like most potential filed methods. Furthermore, the gradient descent method can only achieve a linear convergence rate. Therefore, using the basic AP method, heterogonous UAVs/UGVs will spend much more time achieving the goal. The improved Newton method employs the quadratic derivation besides single gradient information, thus it can greatly alleviate oscillations with the quadratic convergence.

In eq. (12), matrix $\boldsymbol{B}$ is based on the improved Newton method. As in the optimization theory, $\boldsymbol{B}$ is a positive-definite matrix and is defined as $\boldsymbol{B}=\left(\boldsymbol{G}+v\boldsymbol{I}\right)^{-1}$. Since the matrix inverse scales with the cube of the dimensionality of the problem, the computational cost for inverting this matrix at every step can become prohibitive [18]. In the navigation problem, this inverse is available in closed form and requires only very modest additional computation. We also present a straightforward extension in the three dimensional space at the expense of little increase of computation. For very high dimensional problems, computational cost may make the improved Newton method unsuitable for real-time navigation.

4.2 Random force scheme

Suppose that the gravitation comes only from the ending point and the gravitation is proportional to the distance and gravity factors. Therefore, at some points near the obstacle, the attractive force can be balanced by the repulsive force, which makes the UAV or UGV unable to continue the travel. A key feature of this problem is the force dose not directly operat on the UAV or UGV, and the gravity chain only plays a guiding role to them. To solve this local optimum problem, we use a random force scheme to motivate the UAV or UGV when the resultant force on it is stable. The random force scheme contains two parts.

1) While an UAV or UGV is not labelled as plunging into a local minimum, do as follows.

a) Record the difference of the UAV or UGV positions $\Delta p_{i+1}=\left\|\boldsymbol{p}_{i+1}-\boldsymbol{p}_i\right\|$ at every sampling time. If its position increment is less than a specified value $\Delta p_{i+1}<m_1$, then the agent's *Bas* value plus one.

b) If *Bas* is larger than the certain threshold *Limit*, the UAV or UGV would be labelled as falling into a local minimum.

2) While an UAV or UGV is labeled as plunging into a local minimum, generate a random control vector $\boldsymbol{u}'$ based on a random force $\boldsymbol{F}_{\text{rad}}$ and add it to the original control input $\boldsymbol{u}$. The magnitude of this force is limited to $[\min(\boldsymbol{F}_{\text{att}},\boldsymbol{F}_{\text{rep}}),\max(\boldsymbol{F}_{\text{att}},\boldsymbol{F}_{\text{rep}})]$, and the direction is perpendicular to $\boldsymbol{F}_{\text{att}}$. As shown in Figure 2, the magnitude of random force is uncertain, but it only has two possible directions.

5 Comparative experimental results

In order to measure the performance of our proposed improved AP method, simulation experiments were carried out in different environments. The algorithm is implemented by Matlab-2009a, and no commercial algorithm tools are used. The initial value of R_{rep}=2, G_{goal}=1000, G_{obs}=100, f=0.2,

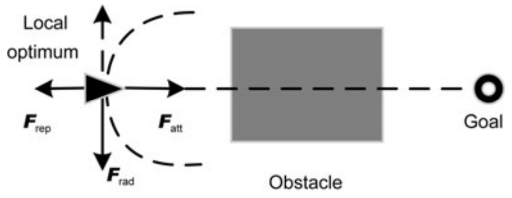

Figure 2 Random force on agent.

Limit=5, m_1=1, the sampling interval Δt =0.1 s, and the maximum velocity for all heterogonous UAVs/UGVs $v_{\max}$= 5 m/s.

In order to demonstrate advantages of the improved AP method in solving local optimum problems, we compared the method with basic AP in an environment with a local optimum. There is a UAV and a UGV in the heterogonous group. The "★" indicates the goal position and the solid cube is the obstacle. The obstacle produces the repulsive force to reject the UAVs/UGVs and the target generates the attractive force.

Figure 3 shows the motion trajectories of a heterogonous UAV and UGV using the basic AP method when moving forward to a cubic obstacle. It can be seen that the agents are suffering from the local minima problem of artificial force field. The UAV or UGV falls into a local minimum region and faileds to get the goal target.

The trajectories of heterogonous UAVs/UGVs generated by the improved AP method in the complicated environment are shown in Figure 4. The path generated by the proposed method successfully navigates the agent to the target and get rid of the local optimum.

We use fixed cubic obstacles in an area (120 m×120 m) to compare the performance of the basic AP, potential field and improved AP methods in the complex environment. This environment represents a complex, unknown, crowded three-dimensional urban environment [19–21]. Figure 5(a) and (b) show the trajectories of the basic AP, potential field and improved AP methods, where the "★" on the upper right represents the goal. The "■" and "●" indicate the UAV and the UGV, respectively. Both the two methods can reach the goal through the complex obstacle area. By using the improved AP method, heterogonous UAVs/UGVs spend much fewer time steps seeking the goal. Otherwise, we can see that a large number of oscillations are caused by the basic AP method when obstacles are nearby.

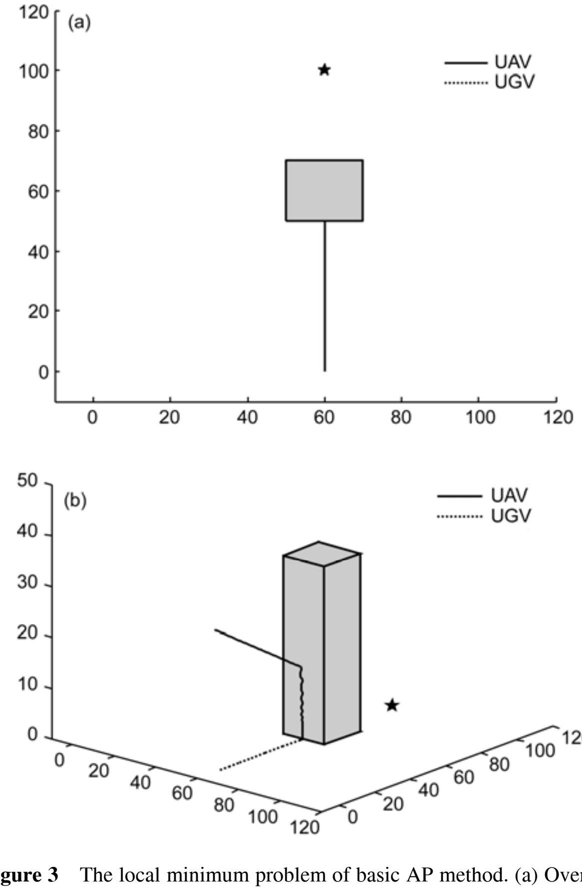

Figure 3 The local minimum problem of basic AP method. (a) Overhead view of the trajectories; (b) three-dimensional view of the trajectories.

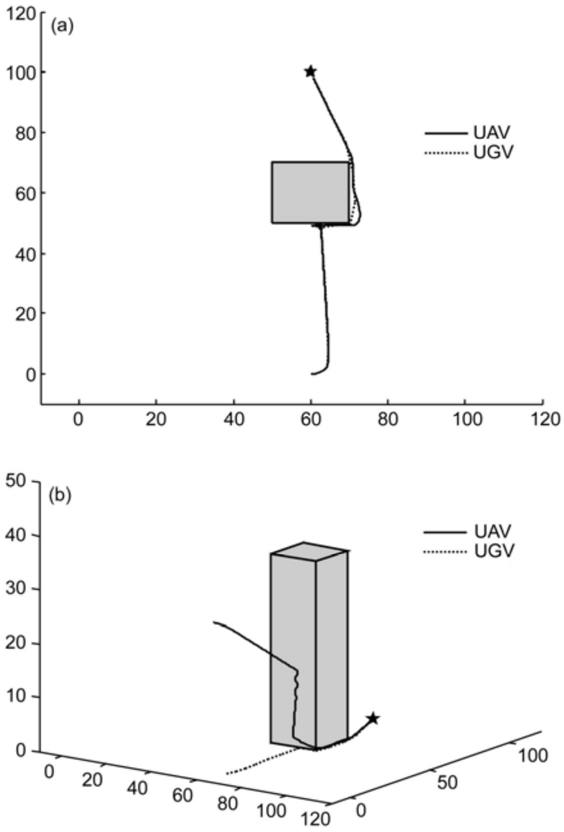

Figure 4 The locus based on the improved AP method, which can effectively solve the local minimum problem. (a) Overhead view of the trajectories; (b) three-dimensional view of the trajectories.

Based on these experimental results and analysis, it is obvious that our proposed improved AP method has advantages over the basic AP method and the general potential field method when applying to the heterogonous multiple UAVs/UGVs navigation in a complex environment. The improved AP method can attain smoother trajectories without serious oscillations and avoid the local optimum. Compared with the general potential field method, the AP method does not need to construct the complicated potential function.

In PFM applications, oscillations depend greatly on the shape of the potential contour. The modified AP method employs the Hessian matrix of corresponding conservative force, which contains quadratic gradient information, thus it

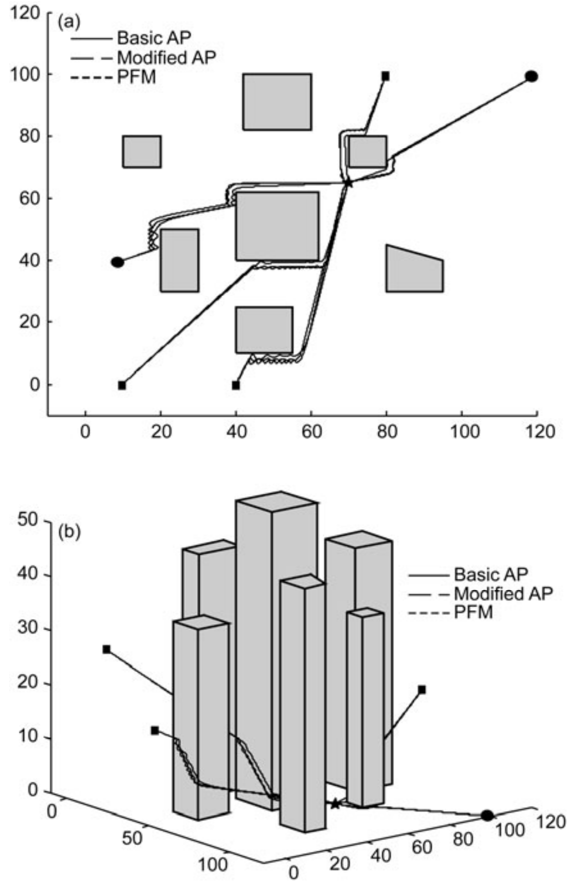

Figure 5 Trajactaries of the heterogonous UAVs/UGVs with multiple cubic obstacles using three methods. (a) Overhead view of the trajectories; (b) three-dimensional view of the trajectories.

can greatly alleviate oscillations with the quadratic convergence. There are other techniques from the optimization theory (such as the conjugate gradient approach) that might be used to eliminate oscillations. Compared with the general PFM, the AP method does not need to construct the complicated potential function but exerts virtual physics forces on the UAVs/UGVs directly, thus it has a more accurate, clear, and comprehensive physical meaning.

6 Conclusions

In this paper, we have proposed a modified AP-based method for the problem of heterogonous UAVs/UGVs coordination, and also presented a random force scheme to solve the local minima problem through complex obstacle fields. Some comparative experiments were conducted by using the basic AP, potential field and improved AP methods. The experimental results demonstrated that our proposed method can greatly improve the overall system performance. Our future work will focus on how to implement our proposed method to real world multiple UAVs/ UGVs systems. Furthermore, we will continue find better solutions by investigating the close connection between physics and distributed control. And bio-inspired computation methods, such as particle swarm optimization [22, 23], differential evolution [24], artificial bee colony optimization, are also promising intelligent approaches for solving such type of complicated problems.

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