# Quantum-behaved Pigeon-inspired Optimization Algorithm based on Particle-best Mutation


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***Abstract*—Aiming at the problem that the Quantum-behaved Pigeon-inspired Optimization Algorithm is easy to fall into the local optimum and is lack of fine search ability, a new algorithm based on particle-best mutation is proposed. Firstly, aiming at the map and compass operator characteristics of solve rough search, Cauthy mutation based on particle-best was adopted for the pigeons when they have not evolved at this stage, so as to enhance the global search ability. Then, aiming at the landmark operator characteristics of solve fine search, Gaussian mutation after updating by landmark operator was adopted for the pigeons when they have not evolved at this stage, so as to enhance the local search ability. Finally, model of pigeons' number factor which likes map and compass factor was built to keep the pigeons in the landmark operator at a certain amount, so as to get the optimal solution. The function test results show that the search speed and accuracy of the improved algorithm are better than other popular algorithms.**



***Keywords—swarm intelligence, pigeon-inspired optimization, Cauchy mutation, Gaussian mutation, particle-best, number factor***


## I. INTRODUCTION

In July 2017, the State Council issued a new generation of artificial intelligence development planning notice, the group intelligence theory as one of the eight basic theories, the key technology of group intelligence as one of the eight key common technologies. In the theory of swarm intelligence, the key points are to break through the theory and methods of organization, emergence and learning of swarm intelligence, and to establish an extensible and computable swarm intelligence incentive algorithm and model. The swarm intelligence algorithm achieves the purpose of optimization by simulating various group behaviors of social animals and utilizing information interaction and cooperation among individuals in the swarm[1][2], such as ant colony algorithm, particle swarm optimization, and mixed leapfrog algorithm, artificial bee colony algorithm, firefly algorithm, pigeon-inspired optimization algorithm(PIO), whale optimization algorithm[3][4]. Among the many intelligent optimization algorithms, the PIO has received much attention since it was proposed by H. B. Duan in 2014[5]. The basic idea comes from the independent homing behavior of the pigeons. The PIO algorithm is simple and easy to implement. It has been successfully applied in many fields such as UAV formation, control parameter optimization, image processing, medical imaging, biological detection, filtering[6]~[8].

H. H. Li[9] proposed a QPIO algorithm, which is inspired by quantum mechanics on the basis of the convergence behavior of a single pigeon, which makes the pigeons have quantum behavior. Its remarkable features are fewer control parameters, simple setup and strong search ability, has a good global search ability. H. R[10] proposed that the map and compass factor undergo nonlinear changes in the iterative process, and introduce the cross-concept of genetic algorithm in PIO algorithm. H. B. Duan[11] proposed a predator escape mechanism to improve the overall performance of the pigeons in order to optimize the basic pigeons. In view of the problem that PIO is easy to fall into local optimum, S. J. Zhang[12] and H. B. Duan[13] used Gaussian pigeon-inspired optimization (CPIO) and Cauthy mutation pigeon inspired optimization (CMPIO) respectively. The pigeons are disturbed, which effectively reduces the probability that the optimization result falls into local optimum.

The above research effectively promotes the development of PIO algorithm, which makes the algorithm have the advantages of fast convergence and high search efficiency. However, the problem of easy premature convergence and insufficient search ability is still unresolved. Aiming at this shortcoming, this paper proposes a Particle-best Mutation Pigeon Inspired Optimization (PMPIO) algorithm. On the basis of maintaining the search ability of the quantum pigeon group optimization algorithm, the algorithm uses different perturbation strategies for the different characteristics of the pigeons in different search stages. At the same time, in the process of evolution of the pigeons, the number of pigeons is kept at a reasonable scale, so that it can enhance the local search ability as much as possible while maintaining rapid convergence.


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## II. PIO ALGORITHM AND QPIO ALGORITHM

### A. PIO algorithm

The PIO algorithm uses two different operator models by simulating the different stages of the pigeon's search for the target, using different navigation tools: map and compass operator, landmark operator. In this model, virtual pigeons are used to simulate the navigation process.

*(1) Map and compass operator*

The position and speed of the *i*-th pigeon are recorded as

$$X_i = [x_{i1}, x_{i2}, \cdots, x_{iD}] \quad V_i = [v_{i1}, v_{i2}, \cdots, v_{iD}]$$

where $i = 1,2,\ldots,N$. $N$ is the group size; $D$ is the search space dimension. Each pigeon updates its position $X_i$ and speed $V_i$ according to Equation (1) and (2).

$$X_i(t) = X_i(t-1) + V_i(t) \tag{1}$$

$$V_i(t) = V_i(t-1)\mathrm{e}^{-R\times t} + \mathrm{rand}(X_{gbest}(t-1) - X_i(t-1)) \tag{2}$$

where $R$ is the map and compass factor. As the iteration proceeds, the flight speed of the pigeon can be reduced; rand is a random number, rand∈[0,1]; $t$ is the current number of iterations; $X_{gbest}(t)$ is the current optimal global position for the search. Is the result of comparison with other pigeons. When the number of iterations reaches the preset number of iterations $N_{c1}$, the map and compass operators are stopped, and the landmark operator is executed to continue execution.

*(2) Landmark operator*

In the landmark operator, the total number of pigeons will be halved after each iteration. The method of updating the quantity is shown in Equation (3).

$$N_p(t) = \frac{N_p(t-1)}{2} \tag{3}$$

$X_c$ is the center position of the remaining pigeons and is used as a landmark, ie as a reference direction for flight. The update method is shown in Equation (4) and (5).

$$X_c(t) = \frac{\sum X_i(t) fitness(X_i(t))}{\sum fitness(X_i(t)} \tag{4}$$

$$X_i(t) = X_i(t-1) + rand(X_c(t) - X_i(t-1)) \tag{5}$$

After the $N_{c2}$ iteration, the global optimal position $P_g$ is obtained.

### B. QPIO algorithm

The Quantum Pigeon Group Optimization Algorithm (QPIO) is a new global optimization algorithm based on the PIO algorithm combined with the motion law of particles in one-dimensional $\delta$ potential well in quantum mechanics [9].

$m_{\mathrm{best}}(t)$ is the average of the pigeons at the *t*-th iteration, indicating the average best position in the population, defined as Equation (6).

$$m_{\mathrm{best}}(t+1) = \frac{1}{N_p} \sum_{i=1}^{N_p} P_i(t) \tag{6}$$

where $P_i(t)$ is the optimal position currently searched for each pigeon.

Create a new subgroup $P_i P_g(t+1)$

$$P_i P_g(t+1) = f(t+1) \times P_i(t) + (1 - f(t+1)) \times P_g(t) \tag{7}$$

where $P_g(t)$ is the optimal position currently searched for the whole pigeon, and $f(t+1)$ is a random number with uniform value in the range of 0~1.

Set $\omega(t)$ be the expansion-contraction factor, which is used to control the convergence speed of the algorithm. The value is the linear function of the number of iterations of the algorithm.

$$\omega(t) = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \times \frac{t}{t_{\max}} \tag{8}$$

where $t_{\max}$ is the total number of iterations, $\omega_{\max}$, $\omega_{\min}$ is a non-negative constant.

The way to update individual pigeons is

$$\begin{cases} X_i(t+1) = P_i P_g(t+1) + \omega(t+1) \times |m_{\mathrm{best}}(t+1) - X_i(t)| \times \ln\frac{1}{u(t+1)}, & f(t+1) \geq 0.5 \\ X_i(t+1) = P_i P_g(t+1) - \omega(t+1) \times |m_{\mathrm{best}}(t+1) - X_i(t)| \times \ln\frac{1}{u(t+1)}, & f(t+1) < 0.5 \end{cases} \tag{9}$$

where $u(t+1)$ is a random number with uniform values in the range of 0~1.

## III. QPIO ALGORITHM BASED ON PARTICLE-BEST MUTATION

### A. Disturbance

Although the quantum pigeon group optimization algorithm is perturbed, the individual searches only in the vicinity of $g_{\mathrm{best}}$ in the global search. For the more complex search space, the group is still easy to fall into the local optimum. That is to say, when the group gathers in a small range, it will lead to the lack of group diversity, which is prone to local convergence in solving the multi-peak optimization problem or "stagnation" in a long period of time. The specific analysis is described.

Equation (9) can be rewritten as Equation (10).

$$X_i(t+1) = P_i P_g(t+1) \pm \omega(t+1) \times |m_{\mathrm{best}}(t+1) - X_i(t)| \times \ln\frac{1}{u(t+1)} \tag{10}$$

Bringing Equation (7) into Equation (10),

$$X_i(t+1) = f(t+1)\left(P_i(t) - P_g(t)\right) + P_g(t) \pm$$

$$\omega(t+1) \times |m_{\mathrm{best}}(t+1) - X_i(t)| \times \ln\frac{1}{u(t+1)} \tag{11}$$

Equation (11) shows that in the search process, if the group moves in a small space, the current position of the pigeon $X_i$, the individual optimal position $P_i$ of the pigeon and the global optimal position $P_g$ are very close. So that the value of the $(P_i(t) - P_g(t))$ term in Equation (11) is very small, even 0. In this way, the effect of the item on the position update of the pigeon is small, which further leads to the lack of group diversity, which weakens the search ability of the group in a large range and falls into local optimum. Therefore, in order to maintain the diversity of the pigeon population throughout the search process, it is necessary to mutate the pigeons.

In evolutionary computation theory, Gaussian mutation and Cauchy mutation are two commonly used mutation operators.

If the Cauchy variation is used, then X is a random variable X=C that satisfies the Cauchy distribution, and its probability density function is shown in Equation (12).

$$f_C(x) = \frac{1}{\pi}\left(\frac{a}{a^2+x^2}\right) \tag{12}$$

where $a$ is the scale parameter. Its variation value is shown in Equation (13).

$$a \times \tan[\pi(\text{rand} - 0.5)] \tag{13}$$

If Gaussian variation is used, then $X$ is a random variable $X$=$N$(0,1) that satisfies the Gaussian distribution, and its probability density function is as shown in Equation (14).

$$f_N(x) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{x^2}{2}\right) \tag{14}$$

Its variation value is shown in Equation (15).

$$b \times \text{randn} \tag{15}$$

Where $b$ is the scale parameter.

The biggest difference between the pigeon herd optimization algorithm and the previous swarm intelligence optimization algorithm is that the search process is divided into two stages and different operator models are used. The first stage map and compass operator are mainly used to guide the pigeons to the destination, which is a rough search. The second stage landmark operator is mainly used to guide the pigeons to the final destination, which belongs to the fine search.

For the different search requirements of the two stages, when selecting the perturbation algorithm, it needs to be considered separately. Cauchy mutations have advantages in jumping out of local optimum, while Gaussian mutations perform better in local convergence [14]. Therefore, for the PIO algorithm, the Cauchy variation can be introduced into the map and the compass operator, and the Gaussian variation can be introduced into the landmark operator. When the first stage rough search falls into the local optimum, the Cauchy variation is used to increase the disturbance range and expand the global search range. When the second-stage fine search falls into local optimum, the Gaussian mutation is used to search in a small range, and the local search range is enlarged to ensure convergence. This can avoid premature convergence and fall into the local optimal problem, and can ensure that the landmark operator finds the global optimal solution.

The mutation operation of the current improved algorithm mostly mutates from the current position of the pigeon. The range of variation is not too large in probability, and belongs to local search [12][13]. However, for the map and compass operators in the rough search phase, the purpose of the search is to guide the pigeons to the best advantage, but not to reach the optimal position. At this time, if the variation is based on the current position of the pigeon, it is very likely that all the pigeons are not in the optimal position. In the minimum problem shown in Figure 1, the global best solution lies in $G$, and $P_i$ is the historical best of individual $i$. The pigeons currently at the three points $A$, $B$ and $C$ are some distance away from the best advantage $G$. If the mutation search is performed near the current position, it may still not be able to jump to the local optimum. Therefore, all the pigeons in this case are gathered near the local best, and never reach the global best. However, the most important advantage of individual history $P_i$ is likely to have been in the vicinity of global optimality. This information should be fully utilized in the dynamic search process.

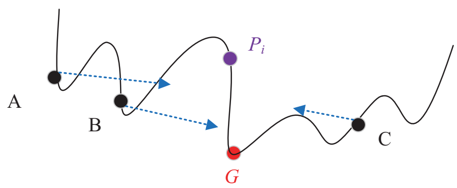


Fig. 1. Local optimal indication

Therefore, in the map and compass operators, if they fall into local optimum, this paper proposes to complete the mutation operation near the optimal position of each pigeon's own search, instead of using the new concept of continuing to search using the previous generation position. The timing of performing the mutation operation is that when the global optimal fitness value is less than Th1 in the continuous $N_{c1\max}$ sub-variation, the Cauchy mutation operation is performed on the optimal position of the individual according to the Cauchy distribution probability density function. The position of the $i$-th pigeon is updated as shown in Equation (16).

$$X_i(t+1) = P_i(t) + V_i(t+1) \tag{16}$$

where $V_i(t+1) = a \times \tan[\pi(\text{rand} - 0.5)]$. That is, the position update of the pigeon is determined by the sum of the individual's optimal position of the pigeon and the Cauchy variation.

This variant applies only to rough searches of maps and compass operators, but not to landmark operators. Because the remaining pigeons that execute the landmark operator are already near the global optimal value, a fine search is needed at this time. That is, the location should not have a large change. Gaussian variation can satisfy the above conditions. Therefore, in the landmark operator, if it falls into local optimum, Gaussian mutation is used to obtain the global optimal value. The timing of performing the mutation operation is that when the global optimal fitness value is less than $Th_2$ in the continuous $N_{c2\max}$ sub-variation, the Gaussian variation operation is performed on the individual position according to the Gaussian distribution probability density function. The position of the $i$-th pigeon is updated as shown in Equation (17).

$$X_i(t+1) = X_i(t) + \text{rand} \times \left(X_c(t) - X_i(t)\right) + G_s \quad (17)$$

where, $G_s$ is a Gaussian variation term whose $k$-th dimension is shown in Equation (18).

$$Gs_k = b \times \text{randn} \quad (18)$$

That is, the position update of the pigeon is determined by the sum of the pigeon landmark operator update result and the Gaussian variance value.

### *B. Pigeon swarm evolution*

The core idea of the introduction of landmark operators into the optimization of pigeons is to enhance the local optimization ability of pigeons and quickly obtain the optimal solution. The premise is that the pigeons quickly move closer to the optimal solution while maintaining a certain population. However, in the landmark operator of the standard pigeon group optimization algorithm, the number of pigeons will be reduced by half after each iteration, that is, exponentially decreasing. The loss of the pigeon population is too large. Regardless of the initial number of pigeons, they will quickly drop to 2 or 1 (the result of rounding up is 2, the result of rounding is 1), and it remains unchanged at 2 or 1. Therefore, the diversity of the algorithm is lost, which greatly affects the optimization performance of the algorithm. Therefore, this paper uses the number of pigeons similar to the map and compass factor to adjust the number of pigeons in the landmark operator, shown in Equation (19).

$$N_p(t) = PigeonNumInit \times \mathrm{e}^{-R \times t} \quad (19)$$

where *PigeonNumInit* is the initial number of pigeons.

At the beginning of the landmark operator, a large number of pigeons are far away from the destination. At this time, the best contribution to the group is small, and it needs a lot of abandonment. Therefore, the number of pigeons is reduced faster. In the later stage of the landmark operator, the remaining pigeons gradually gather near the destination, and it is necessary to increase the influence on the central position while maintaining a certain population size. Therefore, the number of pigeons is reduced at a slower rate. This also helps the convergence of the algorithm.

The computational cost of various types of intelligent optimization algorithms is mainly due to the complexity of the objective function itself and the number of evaluations [15]. The improved algorithm does not significantly increase the amount of computation of the objective function.

## IV. NUMERICAL EXPERIMENTS

### *A. Benchmark functions and other algorithms for comparison*

In order to test the performance of the algorithm, this paper uses the standard test functions of Reference [9]: Shubert function, Rosenbrock function, Rastrigin function and Schaffer function.

Simulation experiments were carried out on three algorithms: QPIO [9], CMPIO [13] and PMPIO. This paper sets the initial population of pigeons *PigeonNumInit*=50, map and compass factor, number of pigeons factor $R$=0.2, number of map and compass operator iterations (ie map and compass operator termination conditions) $N_{c1}$=50, number of landmark operator iterations ( That is, the landmark operator termination condition) $N_{c2}$=10, the Cauchy mutation condition $N_{c1max}$=3, the Gaussian variation condition $N_{c2max}$=2, the map and compass operator variation threshold $Th_1$=0.1, and the landmark operator variation threshold $Th_2$=0.001.

### *B. Experimental results and analysis*

The Monte Carlo simulations were performed on these four test functions respectively. The statistical results of each algorithm for different function optimization are shown in Table I~IV.

According to the experimental results, it can be found that although the three PIO algorithms tested can obtain the optimal solution, the number of iterations and the refined search ability required by each algorithm in obtaining the optimal solution are significantly different. Due to the addition of Cauchy disturbances in the map and compass operators, the PMPIO algorithm significantly speeds up the search for the vicinity of the optimal solution; due to the addition of Gaussian perturbations to the landmark operators, the pigeons are able to perform detailed searches near the optimal values. At the same time, the number of pigeons is used to maintain a certain size of the population, so that the optimal value is better than the other two algorithms.

## V. CONCLUSION

This paper proposes a quantum pigeon group optimization algorithm based on individual optimal variation. The difference between the algorithm and the traditional PIO algorithm is that, on the one hand, different mutation perturbation methods are used for the different task characteristics of the map and the compass operator and the landmark operator, so that the pigeons can quickly and roughly search for the vicinity of the global optimal value. Fine search; on the other hand, using the number of pigeons similar to the map and compass factor, adjust the number of pigeons in the landmark operator to ensure the diversity of the pigeons in the fine search, so that the optimal value can be searched.

In the next step, the pigeon breeding optimization algorithm will be further studied in terms of boundary processing and the processing of the center position of the remaining pigeons, which will speed up the search and study the optimization problems under discrete conditions and apply them to solve practical problems.

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TABLE I. STATISTICAL RESULTS OF EACH ALGORITHM IN THE OPTIMIZATION OF SHUBERT FUNCTION

| Algorithm | Optimal value | Worst value | Mean value | Standard deviation |
|---|---|---|---|---|
| QPIO | -1.867308e+02 | -1.795573e+02 | -1.864226e+02 | 8.082380e-01 |
| CMPIO | -1.867309e+02 | -1.851608e+02 | -1.866043e+02 | 2.680723e-01 |
| PMPIO | -1.867309e+02 | -1.864336e+02 | -1.867119e+02 | 4.479467e-02 |

TABLE II. STATISTICAL RESULTS OF EACH ALGORITHM IN THE OPTIMIZATION OF ROSENBROCK FUNCTION

| dimension | Algorithm | Optimal value | Worst value | Mean value | Standard deviation |
|---|---|---|---|---|---|
| | QPIO | 1.536770e-13 | 6.327500e-03 | 2.975508e-04 | 1.089167e-03 |
| 10 | CMPIO | 3.532382e-09 | 5.312296e-04 | 2.725337e-05 | 6.098332e-04 |
| | PMPIO | 2.135578e-08 | 8.079711e-05 | 7.612065e-06 | 1.402009e-05 |
| | QPIO | 4.927919e-12 | 2.364817e-02 | 6.098172e-04 | 2.937569e-03 |
| 20 | CMPIO | 2.875068e-08 | 6.342611e-03 | 3.404107e-04 | 9.535897e-03 |
| | PMPIO | 6.971481e-08 | 9.898792e-04 | 6.950810e-05 | 1.363482e-04 |
| | QPIO | 7.384892e-14 | 3.043208e-02 | 6.609397e-04 | 3.429567e-03 |
| 30 | CMPIO | 7.534186e-07 | 4.263443e-03 | 4.573720e-04 | 8.435884e-03 |
| | PMPIO | 2.378869e-08 | 4.375013e-04 | 7.480064e-05 | 9.323265e-05 |

TABLE III. STATISTICAL RESULTS OF EACH ALGORITHM IN THE OPTIMIZATION OF RASTRIGIN FUNCTION

| dimension | Algorithm | Optimal value | Worst value | Mean value | Standard deviation |
|---|---|---|---|---|---|
| | QPIO | 0.000000e+00 | 3.656290e-08 | 1.080452e-08 | 4.364021e-08 |
| 10 | CMPIO | 3.599036e-10 | 9.158256e-08 | 1.596909e-09 | 1.838259e-09 |
| | PMPIO | 0.000000e+00 | 1.243450e-14 | 2.131628e-16 | 1.389330e-15 |
| | QPIO | 9.947598e-14 | 1.809783e-08 | 1.745212e-08 | 4.056805e-08 |
| 20 | CMPIO | 1.159227e-12 | 3.270251e-07 | 3.619590e-08 | 5.235973e-07 |
| | PMPIO | 0.000000e+00 | 1.900702e-13 | 2.629008e-15 | 1.994512e-14 |
| | QPIO | 1.048051e-13 | 1.287680e-07 | 2.712775e-09 | 1.329087e-08 |
| 30 | CMPIO | 1.729265e-12 | 5.977787e-07 | 4.668195e-08 | 7.632764e-07 |
| | PMPIO | 0.000000e+00 | 3.298695e-12 | 4.892087e-14 | 3.417617e-13 |

TABLE IV. STATISTICAL RESULTS OF EACH ALGORITHM IN THE OPTIMIZATION OF SCHAFFER FUNCTION

| Algorithm | Optimal value | Worst value | Mean value | Standard deviation |
|---|---|---|---|---|
| QPIO | 0.000000e+00 | 1.558070e-03 | 3.223010e-06 | 1.636524e-05 |
| CMPIO | 4.216066e-14 | 1.726083e-04 | 4.305834e-11 | 2.111004e-10 |
| PMPIO | 0.000000e+00 | 3.458345e-14 | 7.238654e-16 | 3.856044e-15 |

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