Gaussian Entropy Weight Pigeon-Inspired Optimization for Rectangular Waveguide Design

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***Abstract*—In this paper, a hybrid model of Entropy Weight Method (EWM) and Gaussian Pigeon-Inspired Optimization (GPIO) is proposed to solve the problem of rectangular waveguide design. There are four parameters involved in the design: length, width, relative permittivity and electrical conductivity. Our goal is to optimize the combination of these parameters so that the waveguide achieves the best overall performance. EWM is employed in the building of the performance judging function. An improved Pigeon-Inspired Optimization adopting Gaussian searching strategy is utilized for minimizing the function's value. Comparative experiments with basic PIO and Particle Swarm Optimization (PSO) are conducted, and the results verify that better design can be obtained with our proposed Gaussian Entropy Weight Pigeon-Inspired Optimization (GEWPIO) algorithm because of its utilization of information theory and non-linear searching strategy.**

***Keywords*—Gaussian Pigeon-Inspired Optimization (GPIO), Entropy Weight Method (EWM), Rectangular waveguide design.**

I. Introduction

Microwave is a form of electromagnetic radiation with frequencies ranging from 300 MHz (100 cm) to 300 GHz (0.1 cm), and in today's society microwave is widely used in many aspects: spacecraft communication, navigation, radar and heating application. Metal pipe waveguides are often used to guide microwave, and rectangular waveguides are one of the most common types [1].

It has always been an important task to design the optimal rectangular waveguide, and a lot of relevant work has been conducted. Schmiedel and Arndt proposed a design method based on field expansion of eigenmodes [2]. M. K. Chin's approach is based on an approximate solution of the wave propagation [3]. However, one of the main defects that traditional mathematical optimization methods face is that their effectiveness decreases as the computation's complexity grows [4].

The concept of entropy was originally a thermodynamic construct, and was firstly introduced into information theory by C.E. Shannon [5]. Entropy can measure not only the disorder degree of a group of molecules, but also the amount of effective information provided by a set of data, which lays the foundation for the entropy weight method. In order to fairly judge the performance of a certain waveguide design, we build an objective function taking different evaluating indexes into consideration and utilize EWM to decide the coefficients of each index.

Over the past decades, researchers have put forward many swarm intelligence algorithms, such as Particle Swarm Optimization (PSO) [6], Ant Colony Optimization (ACO) [7], and Artificial Bee Colony (ABC) Optimization [8] to solve complex problems. Pigeon-Inspired Optimization (PIO) is a novel swarm intelligence algorithm firstly proposed by Duan in 2014 [9], and fast convergence speed is its most outstanding advantage. However, PIO can easily trap into a local optimal solution because of the inherent weakness of its uniform distribution random searching system. To overcome this weakness, an improved PIO algorithm adopting Gaussian strategy is proposed and employed in minimizing the objective function's value. In this paper, we propose an intelligent approach combining Entropy Weight Method (EWM) and Gaussian Pigeon-Inspired Optimization (GPIO) to solve the waveguide design problem.

The remaining part of this paper is organized as follows. In section 2, the rectangular waveguide design problem is formulated, where some important parameters and the evaluating indexes are also discussed. In section 3, the main idea of EWM and its detailed implementation steps are introduced. Section 4 describes the basic PIO and our modified version respectively, and section 5 contains the results and analysis of a series of comparative experiments. In the last part, our concluding remarks are provided.

II. Problem Formulation

The shape of a rectangular waveguide is as shown in Fig.1. The length of its cross-section is denoted by a, and the width is denoted by b. The walls are made of metal with electrical conductivity σ and magnetic permeability μ, and material with relative permittivity ε_r fills the inside of the waveguide.

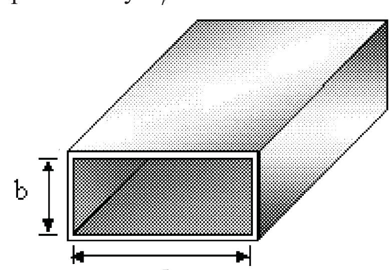

Figure 1. Schematic illustration of rectangular waveguide

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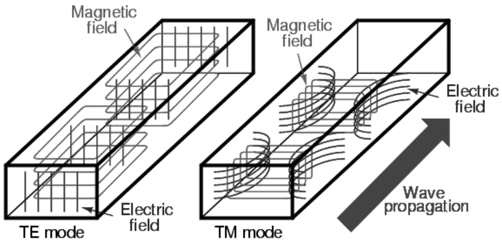


Figure 2. Schematic illustration of TE and TM modes

Microwave propagates through the waveguide by different modes [10], which can be classified into three categories: traverse electric mode (TE), traverse magnetic mode (TM), and traverse electromagnetic mode (TEM). A rectangular waveguide supports TM and TE modes but not TEM, as shown in Fig.2. According to the theory of electromagnetic fields, microwaves cannot propagate beyond a wavelength and this wavelength is called the cut-off wavelength. For a rectangular waveguide, the cut-off wavelength is given by:

$$(\lambda_c)_{mn} = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}} \quad (1)$$

In most realistic cases, single mode operation is preferred, which means that only the dominant mode propagates in the waveguide and all other modes are cut off. To achieve the single mode operation, the following equation should be satisfied:

$$(\lambda_c)_{H10} = 2a > \lambda > \max \begin{cases} (\lambda_c)_{H20} = a \\ (\lambda_c)_{H01} = 2b \end{cases} \quad (2)$$

In order to fairly judge the performance of the rectangular waveguide design, we introduce four evaluating indexes, which will be discussed respectively below.

1. Attenuation Rate

When microwaves propagate in lossy waveguides, the electric field intensity and magnetic field intensity attenuate along the waveguides. To simplify the analysis of wave attenuation, an approximate power-loss method [11] is adopted. In this method, the field expressions are derived assuming perfectly conducting walls, and it gives reasonably accurate result:

$$R = \frac{1}{b}\sqrt{\frac{\pi f_c \varepsilon_0 \varepsilon_r}{\sigma\left[(f/f_c)^2 - 1\right]}}\left[\left(\frac{f}{f_c}\right)^{3/2} + 2\left(\frac{b}{a}\right)\left(\frac{f}{f_c}\right)^{-1/2}\right] \quad (3)$$

2. Power Capacity

The waveguide's power capacity is limited by the phenomenon of electrical breakdown, which is a reduction in the resistance of an electrical insulator when the voltage applied across it exceeds the breakdown voltage. The relationship between power capacity and breakdown electric intensity is given by:

$$P = \frac{ab}{480\pi} E_b^{\,2} \varepsilon_r^{\,2} \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2} \quad (4)$$

3. Band Width

Band width is defined as the difference between the highest frequency signal component and the lowest frequency signal component, and the capacity of a given communication channel is mainly determined by its band width. Band width can be calculated by the following equations:

$$\begin{cases} f_l = c\big/\left(2a\sqrt{\varepsilon_r}\right) \\ f_h = \min\left\{c\big/\left(a\sqrt{\varepsilon_r}\right), c\big/\left(2b\sqrt{\varepsilon_r}\right)\right\} \\ W = f_h - f_l \end{cases} \quad (5)$$

4. Cost of Materials

The cost of materials is of course another factor that should be taken into consideration in the design process, and it can be easily seen: $C \propto 2(a+b)$.

III. ENTROPY WEIGHT METHOD

In section 2, all of the four evaluating indexes are already discussed, and now we build the objective function:

$$F = w_1 f(R) + w_2 f(P) + w_3 f(W) + w_4 f(C) \quad (6)$$

where w_1, w_2, w_3 and w_4 are weight coefficients, and their value will be determined by Entropy Weight Method. According to Shannon's information theory, which was first proposed in his 1948 paper "A Mathematical Theory of Communication", the information entropy can explicitly be written as:

$$H(X) = -\sum_{i=1}^{n} P(x_i)\log_b P(x_i) \quad (7)$$

From this equation we can see that when the values of evaluated objects on a certain index are distributed unevenly, the corresponding entropy is small. Small entropy means that a larger amount of effective information is provided, so the corresponding coefficient should be bigger. On the contrary, the smaller the difference, the larger the entropy is, which means that only a small amount of information is provided and the corresponding coefficient should be smaller. The detailed procedure of using EWM to determine the coefficients is as follows:

1. Standardize original data matrix

Consider a case when there are n evaluated objects and m evaluating indexes, the original data can be written as the following matrix:

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

To standardize the matrix, the result is $(x'_{ij})^{m\times n}$ where x'_{ij} is the standard value of the j_{th} evaluated object on the i_{th} evaluating index, and it is calculated by:

$$x'_{ij} = \left(x_{ij} - \min x_j\right) \Big/ \left(\max x_j - \min x_j\right) \tag{8}$$

In our case of the rectangular waveguide design, there are four evaluating indexes. We choose a set of combinations of parameters and calculate corresponding indexes' value. The orthogonal strategy [12] is introduced to guarantee the diversity of the combinations, and the results after standardization are listed in Table 1.

2. Calculate entropy

According to (7), entropy values of the four evaluating indexes are calculated and the results are listed in Table 2.

3. Calculate weights

After the entropy of the i_{th} index is calculated, the definition of its entropy weight is:

$$w_i = \left(1 - H_i\right) \Big/ \left(m - \sum_{i=1}^{m} H_i\right) \tag{9}$$

where m is the number of indexes. Now that we get the coefficients of the indexes, (6) can be rewritten as:

$$F = 0.246 f(R) + 0.328 f(P) + 0.303 f(W) + 0.124 f(C) \tag{10}$$

The process of choosing the optimal waveguide design is equal to minimizing the objective function's value, which will be addressed in next section.

TABLE I. VALUE OF INDEXES AFTER STANDARDIZATION

a	b	ε_r	σ	R	P	W	C
0.16	0.04	1	6.17	0.9496	1.0000	0.0000	0.00
0.16	0.08	5	3.82	0.7481	0.4450	0.0000	0.25
0.16	0.12	10	1.57	1.0000	0.2600	1.0000	0.50
0.20	0.04	5	1.57	0.7985	0.3572	0.1250	0.25
0.20	0.08	10	6.17	0.1343	0.1236	0.1250	0.50
0.20	0.12	1	3.82	0.1266	0.0458	0.4375	0.75
0.24	0.04	10	3.82	0.3307	0.2199	0.2500	0.50
0.24	0.08	1	1.57	0.0430	0.0000	0.2500	0.75
0.24	0.12	5	6.17	0.0000	0.0000	0.2500	1.00

TABLE II. ENTROPY VALUES

$H(R)$	$H(P)$	$H(W)$	$H(C)$
0.8085	0.7443	0.7642	0.9034

TABLE III. WEIGHTS OF INDEXES

w_1	w_2	w_3	w_4
0.2456	0.3280	0.3025	0.1239

IV. PIGEON-INSPIRED OPTIMIZATION

A. Basic PIO

PIO algorithm is a novel swarm intelligence optimizer invented by Duan, and it solves problems by imitating pigeons' homing behavior. A lot of biological research has revealed that it is with the help of magnetic field, the sun, and landmarks, that pigeons can easily find their home [13]. Pigeons can sense the geomagnetic field to shape the map in their brains [14], and they adjust the direction according to the altitude of the sun. As they fly close to their destination, they rely less on the sun and magnetic field, and landmarks play a more important role.

Inspired by these facts, two operators are introduced in the PIO algorithm: the map and compass operator and the landmark operator, of which the former is designed according to the contribution of the magnetic field and the sun, and the latter is based on the utilization of landmarks. The process of optimization can be seen as the pigeons' homing behavior, and the position of each pigeon is a possible solution, which corresponds to the value of the objective function.

- ***Map and compass operator***

First, each pigeon X_i with initial velocity V_i is randomly initialized within the solution space, and they are denoted as $X_i = [X_{i1}, X_{i2}, \cdots, X_{im}]$, $V_i = [V_{i1}, V_{i2}, \cdots, V_{im}]$, where i is the i_{th} pigeon, and m is the dimension of solution space. The new position X_i and velocity V_i of pigeon i at the t_{th} iteration can be calculated by:

$$\begin{cases} V_i(t) = V_i(t-1) \cdot e^{-Rt} + rand \cdot \left(X_g - X_i(t-1)\right) \\ X_i(t) = X_i(t-1) + V_i(t-1) \end{cases} \tag{11}$$

where R is the map and compass factor introduced to slow down the initially randomized search speed of pigeons; X_g is the global optimal solution; rand is a random number uniformly distributed between 0 and 1.

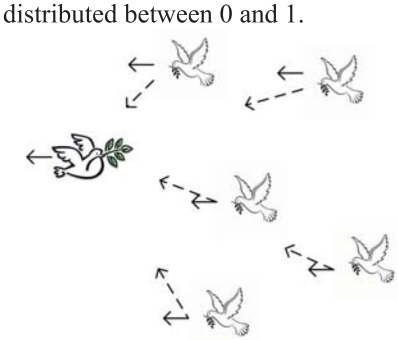


Figure 3. Map and compass operator in the PIO algorithm

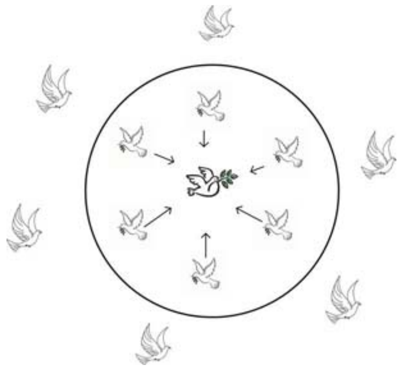


Figure 4. Landmark operator in the PIO algorithm

- ***Landmark operator***

In each generation of landmark operator, the number of pigeons will be halved, which means that pigeons which are far away from the destination will follow those close to the destination, and the number of pigeons at the t_{th} generation is calculated by:

$$N_p(t) = ceil\left(\frac{N_p(t-1)}{2}\right) \quad (12)$$

It is supposed that each pigeon will fly straight to the destination, which in Duan's model [15] refers to the center of all the pigeons in the t_{th} iteration, so the position of pigeon i at the t_{th} iteration is updated by the following equation:

$$X_i(t) = X_i(t-1) + rand \cdot \left(X_c(t) - X_i(t-1)\right) \quad (13)$$

where $X_c(t)$ denotes the center position at the t_{th} iteration, and it is defined as:

$$X_c(t) = \frac{\sum X_i(t) \cdot fitness\left((X_i(t))\right)}{\sum fitness\left(X_i(t)\right)} \quad (14)$$

B. Gaussian PIO

PIO has a certain possibility of trapping into a local optima, and this weakness limits its further application in many situations. To overcome this defect, we formulate a modified PIO model with a Gaussian item added.

In the map and compass operator of basic PIO, the random number obeys uniform distribution. Gaussian distribution is another kind of distribution extensively used in natural science [16], and it can depict many non-linear phenomena better than uniform distribution. Noticing that the searching equation in the map and compass operator satisfies the latent premise of Gaussian distribution, we propose the improved velocity updating equation as follows:

$$\begin{cases} V_i(t) = V_i(t-1) \cdot e^{-Rt} + rand \cdot \left(X_g - X_i(t-1)\right) & if\,(R_2 > p) \\ V_i(t) = V_i(t-1) + R_1 \cdot \left(X_p(t-1) - X_i(t-1)\right) & if\,(R_2 \le p) \end{cases} \quad (15)$$

where R_1 is a random number generated by standard Gaussian distribution, R_2 follows uniform distribution, and p is a flexible parameter used to balance the Gaussian distribution and uniform distribution.

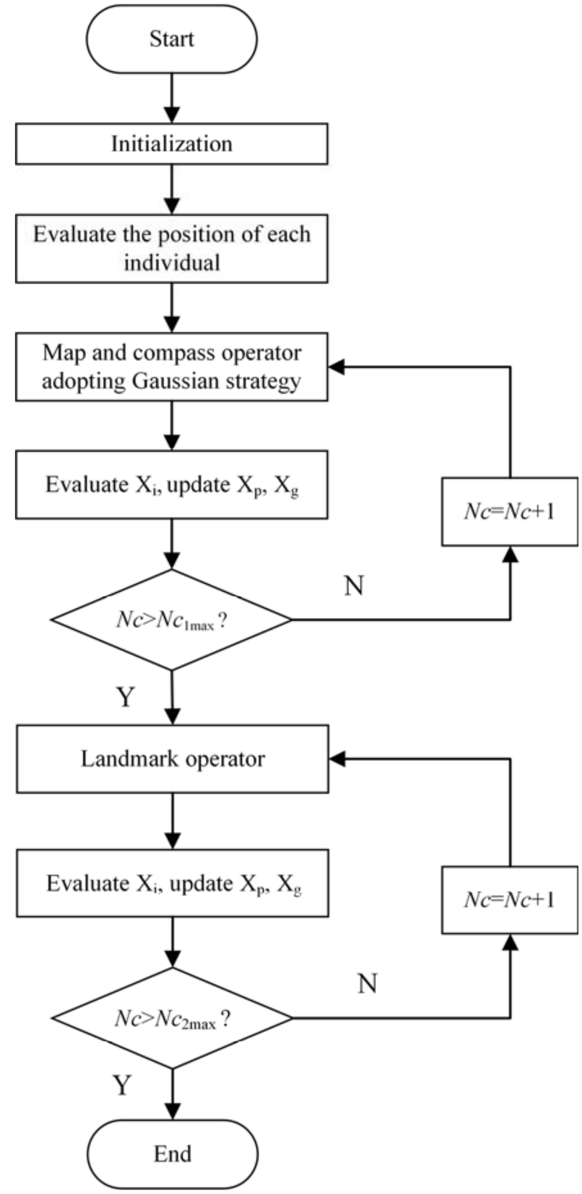


Figure 5. Flow chart of Gaussian PIO

V. Experimental Results

In order to confirm the effectiveness of the proposed GEWPIO algorithm for rectangular waveguide design, we conduct a series of experiments and compare the results with that of basic PIO and Particle Swarm Optimization (PSO). As already discussed, there are four parameters involved in the design: length, width, relative permittivity and electrical conductivity, and our goal is to optimize the combination of these parameters so that the waveguide achieves the best overall performance, which equals that the objective function achieves the minimum value.

We set $\lambda = 0.3m$, and the search space of a and b is decided by (2); the search space of ε_r is between 1 and 10. To simplify the problem, only four kinds of metal commonly used are taken into consideration, and their electrical conductivity value is listed in Table 4.

TABLE IV. CONDUCTIVITY VALUE OF FOUR KINDS OF META.L

Tin	Copper	Aluminum	Silver
0.71×10^7	1.57×10^7	3.82×10^7	6.17×10^7

TABLE V. COMPARATIVE RESULTS OF PSO, PIO AND GEWPIO

	Algorithms	a	b	ε_r	fitness
Tin	GEWPIO	0.1222	0.0609	6.0189	0.7762
	PIO	0.1293	0.0647	5.3800	0.7929
	PSO	0.1255	0.0627	5.7177	0.8034
Copper	GEWPIO	0.1433	0.0717	4.3804	0.6735
	PIO	0.1361	0.0680	4.8593	0.6810
	PSO	0.1310	0.0651	5.2195	0.7036
Aluminum	GEWPIO	0.1195	0.0597	6.3067	0.5836
	PIO	0.1137	0.0569	6.9557	0.6024
	PSO	0.1148	0.0567	6.7735	0.6112
Silver	GEWPIO	0.1103	0.0551	7.4038	0.5494
	PIO	0.1084	0.0542	7.6639	0.5523
	PSO	0.1095	0.0541	7.4120	0.5680

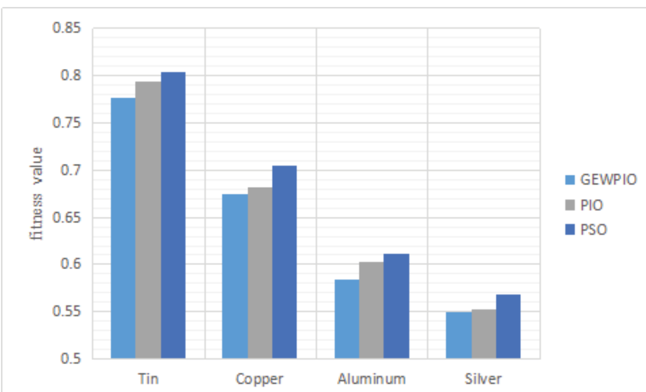
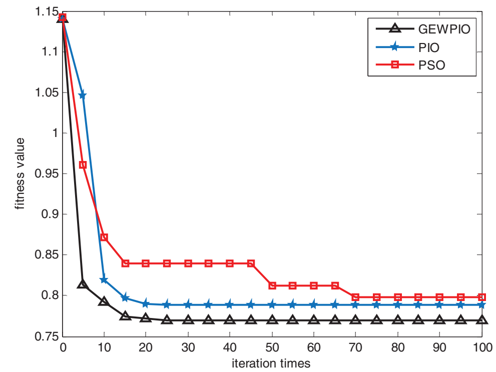


Figure 6. Fitness value for GEWPIO, PIO, and PSO in four cases

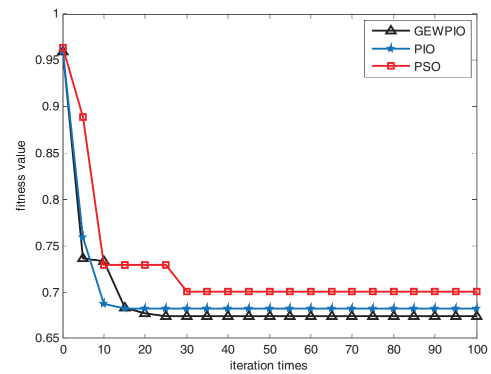
For PSO, we set the number of particles $N=50$, maximum iteration times $T=100$, inertia weight is 0.4, two evolution parameters $C_1=C_2=2$; for PIO and GEWPIO, $T_1=80$, $T_2=100$, $N=50$, $R=0.2$, $p=0.4$. The results are listed in Table 5, and the comparison of fitness value is shown in Fig.6.

It is obvious that the best results are from the GEWPIO algorithm in all of the four cases. We observe that the length is always nearly twice the width. In fact, rectangular waveguides with this configuration are called standard waveguides and are widely used in real practice. To further compare the GEWPIO with the other two algorithms, the evolution curves of the function's fitness value in 100 iteration times for PSO, PIO and GEWPIO are shown in Fig.7.

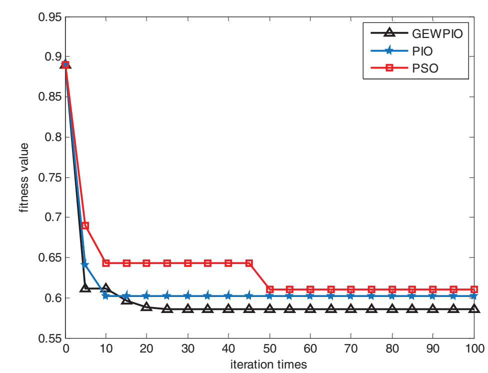
From Fig.7, we can see clearly that GEWPIO has a faster convergence speed compared with PIO and PSO, which means that it can search with higher efficiency.



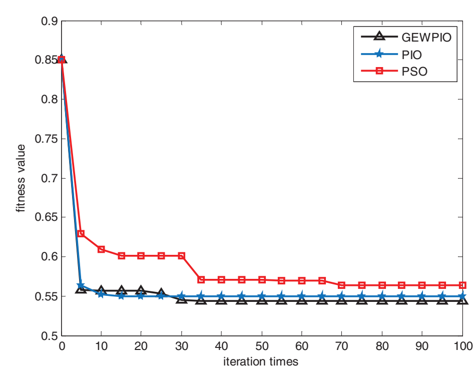
(a) Case of tin



(b) Case of copper



(c) Case of aluminum



(d) Case of silver

Figure 7. Comparative evolution curves of the fitness value for GEWPIO, PIO and PSO in four cases

VI. CONCLUSIONS

In this paper, we combine the Entropy Weight Method and Gaussian PIO to accomplish the task of rectangular waveguide design. EWM is used in the building of the objective function to decide the evaluating indexes' coefficients, and GPIO is employed to minimize the function's value. We conduct a series of experiments, and the results compared with that of basic PIO and Particle Swarm Optimization verify the effectiveness of the proposed Gaussian Entropy Weight Pigeon-Inspired Optimization (GEWPIO) algorithm for rectangular waveguide design. Moreover, the results confirm that in our hybrid GEWPIO model, the utilization of information theory guarantees that the objective function is established reasonably, and the adopted non-linear Gaussian strategy improves the searching efficiency of basic PIO.

REFERENCES

[1] S. F. Mahmoud and J. R. Wait, "Theory of wave propagation along a thin wire inside a rectangular waveguide." *Radio Science* 9.3 (1974): 417-420.

[2] H. Schmiedel and F. Arndt, "Field theory design of rectangular waveguide multiple-slot narrow-wall couplers," *IEEE Transactions on Microwave Theory & Techniques* 34.7(1986):791-798.

[3] M. K. Chin and S. T. Ho, "Design and modeling of waveguide coupled single mode microring resonator," *IEEE Journal of Lightwave Technology* 16.8 (1998):1433-1446.

[4] T. Jansen and F. Neumann, "Computational complexity and evolutionary computation," *Conference Companion on Genetic and Evolutionary Computation* 1972:389-396.

[5] C. E. Shannon, "A mathematical theory of communication," *ACM Sigmobile Mobile Computing and Communications Review* 5.1(2001):3-55.

[6] D. Karaboga and B. Basturk, "A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm," *Journal of Global Optimization* 39.3(2007):459-471.

[7] M. Dorigo and G. D. Carol, "Ant colony optimization: a new meta-heuristic," *Congress on Evolutionary Computation*, 1999, 2(4):1470--1477

[8] J. Kennedy and R. Eberhart, "Particle swarm optimization," I*EEE International Conference on Neural Networks*, 1995, 4(8):1942-1948 vol.4.

[9] H. B. Duan and P. X. Qiao, "Pigeon-inspired optimization: a new swarm intelligence optimizer for air robot path planning," *International Journal of Intelligent Computing and Cybernetics*, 2014, 7(1):24-37.

[10] W. Wang, L. Yue and G. Zhao, "Discrimination and analysis of microwave modes in high power systems," *International Journal of Infrared and Millimeter Waves*, 2005, 26(2):147-161.

[11] J. J. Gustincic, "A general power loss method for attenuation of cavities and waveguides," *IEEE Transactions on Microwave Theory and Techniques* 62.1(1962):20-21.

[12] Q. Feng, S. Liu and J. Zhang, "Extrapolated particle swarm optimization based on orthogonal design," *Journal of Convergence Information Technology,* 2012, 7(2):141-152.

[13] A. Whiten, "Operant study of sun altitude and pigeon navigation," *Nature*, 1972, 237(5355):405-406.

[14] W. T. Keeton, "Magnets interfere with pigeon homing," *Proceedings of the National Academy of Sciences,* 1971, 68(1):102-6.

[15] H. Sun and H. B. Duan, "PID controller design based on Prey-Predator Pigeon-Inspired Optimization algorithm," *IEEE International Conference on Mechatronics and Automation,* 2014:1416-1421.

[16] T. Taguchi, "On a generalization of Gaussian distribution," *Research in Developmental Disabilities*, 1978, 30(1):211-242.