Trajectory Design of Re-entry Vehicles using combined Pigeon Inspired Optimization and Orthogonal Collocation method

Gangireddy Sushnigdha * Ashok Joshi *

* Indian Institute of Technology, Bombay, 400076 India (e-mail: sushnigdha.g@iith.ac.in, ashokj@aero.iitb.ac.in)

Abstract: This paper presents an orthogonal collocation based entry trajectory solution strategy using Pigeon Inspired Optimization (PIO). PIO is a swarm algorithm based on the homing behavior of pigeons. For the unpowered re-entry vehicle, bank angle modulation is considered as the primary control with a predefined nominal angle of attack profile. In this approach, bank angle is approximated using a higher order polynomial. This control variable is discretized at Chebshev Lobatto collocation points. The value of bank angle at these points is obtained using PIO algorithm. The terminal constraints of entry trajectory are part of the objective function. After generating the entry trajectory referred to as reference trajectory, Linear Quadratic Regulator (LQR) control is used to track the reference trajectory for dispersions in the initial states at entry interface. Advantages of PIO algorithm are that it does not require an initial guess and that inequality constraints can be incorporated. This method is demonstrated by applying it to Common Aero Vehicle (CAV-H) with high lift to drag ratio.

© 2018, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Re-entry trajectory optimization, Pigeon inspired optimization, LQR control.

1. INTRODUCTION

Trajectory generation of a re-entry vehicle is regarded as a challenging task because of its highly nonlinear, uncertain entry dynamics with limited control capability and the entry corridor restrictions. Due to the structural limitations of the vehicle, the constraints on allowable heat rate, load factor, and dynamic pressure are to be strictly met. These allowable limits form entry corridor. The entry flight is unpowered with bank angle modulation as the primary control variable.

Entry trajectories are mostly generated offline and are preloaded before the launch. These reference trajectories are tracked onboard using control techniques. The reference longitudinal entry trajectory for the Space Shuttle is determined from the drag-vs-velocity profile and it is tracked onboard using gain scheduling tracking law as reported by Harpold and Graves (1979). Roenneke and Markl (1994), have presented a linear control law that achieves local tracking of drag-vs-energy based reference profile. Dukeman (2002) has applied linear quadratic regulator theory to track the reference profiles of range-to-go, altitude and flight-path angle.

Entry trajectory design is an optimal control problem which is solved through indirect or direct methods. Indirect methods analytically construct the necessary and sufficient conditions for optimality and then solve them numerically as mentioned by Kelly (2017). However, direct methods discretize the control time histories and/or state variable time history, thereby transforming the optimal control problem to a non-linear programming problem (NLP) as defined by Betts (2001). This conversion process is known as transcription which is referred to as direct collocation methods. In the direct collocation method, the control and/or state variables are discretized by approximating all the continuous functions as polynomial splines as stated by Kelly (2017). The Orthogonal collocation method refers to direct collocation method which approximates the continuous functions by higher order orthogonal polynomials as described by Kelly (2017). The pseudospectral methods approximate the entire trajectory using a single higher order polynomial. Pseudospectral methods are used extensively to solve trajectory optimization problems. Legendre pseudospectral method approach is implemented for entry trajectory optimization by Tian and QunZong (2011). Other pseudospectral methods like Gauss and Chebyshev are applied for generating entry trajectories by Jorris et al. (2008) and Cai et al. (2015) respectively. However, pseudospectral methods have tuning issues, as they involve careful selection of basis functions and the number of collocation points.

The evolutionary algorithms that fall under the category of direct methods and mimic natural phenomena have gained popularity because of their simplicity in the problem formulation and nature of having a derivative-free mechanism for solving optimization problems. Genetic algorithm (GA) proposed by Deb (1999), algorithms based on swarm intelligence like Particle Swarm Optimization (PSO) proposed by Eberhart and Kennedy (1995) are some of the popular algorithms. PSO is used to generate optimal trajectories.
for a spacecraft by Rahimi et al. (2013). An evolutionary algorithm named Pigeon Inspired Optimization (PIO) that mimics the homing behavior of pigeons has been proposed by Duan and Qiao (2014). Sushnigdha and Joshi (2017) have converted the problem of finding optimal bank angle profile to single parameter search problem and have solved it using PIO algorithm. Artificial bee colony optimization is used to find the control variable at Legendre Gauss Lobatto collocation points to get the complete optimal control profile for generating entry trajectory by Duan and Li (2015).

In this paper, bank angle is used as the control variable. The paper incorporates the idea of solving orthogonal collocation problem using an evolutionary algorithm. The control profile is assumed to be a continuous function which is approximated by higher order polynomial and discretized at Chebyshev-Lobatto (CL) points. PIO algorithm is used to find the bank angle at these points that minimize the objective function and uses Lagrange polynomial for interpolation. After obtaining the complete optimal bank angle profile, equations of motion are integrated to generate entry trajectories. In order to track some of the profiles under dispersed initial conditions Linear Quadratic Regulator (LQR) tracking law is used. The proposed algorithm is applied to Common Aero Vehicle (CAV-H). Simulation results show that constraints are met accurately. This paper is organised as follows. Section 2 describes the entry dynamics and the constraints involved. Problem statement along with objective function is defined in section 3. Section 4 describes the orthogonal collocation method. Section 5 discusses PIO algorithm. Section 6 gives description of LQR tracking law, and section 7 gives results for validating the presented algorithm. Section 8 concludes the paper.

2. ENTRY VEHICLE DYNAMICS

The re-entry vehicle is assumed to be point mass, gliding over a spherical, rotating Earth whose entry dynamics are given below

\[
\ddot{V} = -g \sin \gamma + \Omega^2 r \cos \phi (\sin \gamma \cos \phi - \cos \gamma \sin \phi \cos \psi) + 2 \Omega V \cos \gamma \sin \psi
\]

\[
\dot{\theta} = \frac{V \cos \gamma \sin \psi}{r \cos \phi}
\]

\[
\dot{\phi} = \frac{V \cos \gamma \cos \psi}{r}
\]

\[
\dot{\gamma} = \frac{1}{V} \left[ L \cos \sigma + (V^2 - g r) \left( \frac{\cos \gamma}{r} \right) + 2 \Omega V \cos \phi \sin \psi \right]
\]

\[
\dot{\psi} = \frac{1}{V} \left[ L \sin \sigma \cos \gamma + \frac{V^2}{r} \cos \gamma \sin \psi \tan \phi - 2 \Omega V (\tan \gamma \cos \psi \cos \phi - \sin \phi) \right. \\
\left. + \frac{\Omega^2 r}{\cos \gamma} (\sin \psi \sin \phi \cos \phi) \right]
\]

\[
\dot{s} = -V \cos \gamma / r
\]

where \( r \) is the radial distance from the Earth center to the vehicle, \( \sigma \) and \( \phi \) are the longitude and latitude, \( V \) is the Earth-relative velocity, \( \gamma \) is the flight-path angle, and \( \psi \) is the heading angle of the velocity vector, measured clockwise in the local horizontal plane from the north as shown in Fig. 1. \( s \) denotes the range to go(in radians) on the surface of a spherical Earth along the great circle connecting the current location of the vehicle and the site of the final destination. \( g = 9.8 \text{m/s}^2 \) is the acceleration due to gravity. The differentiation in the previous equations is with respect to time \( t \). The terms \( L \) and \( D \) are the aerodynamic lift and drag acceleration, respectively.

\[
L = \frac{1}{2m} \rho V^2 C_L S_{ref}
\]

\[
D = \frac{1}{2m} \rho V^2 C_D S_{ref}
\]

The aerodynamic coefficients \( C_L \) and \( C_D \) are functions of angle of attack \( \alpha \) and Mach number. The bank angle \( \sigma \) is defined as the clockwise positive rotation of the lift vector about the velocity vector. \( \Omega \) is the angular velocity of the Earth.

2.1 Path constraints

To ensure the structural safety of the vehicle, entry trajectory should be within the entry corridor formed by the allowable limits on maximum heat rate \( \dot{Q}_{max} \) on surface of the vehicle, load factor \( a_{max} \) and dynamic pressure \( p_{max} \) as given by (10), (11), (12) respectively and also the entry trajectory has to be smooth. The above mentioned constraints form the path constraints that are to be strictly satisfied.

\[
\dot{Q} = 9.4369 \times 10^{-5} \sqrt{V}^{3.15} \leq \dot{Q}_{max}
\]

\[
a = \sqrt{L^2 + D^2} \leq a_{max}
\]

\[
p = (\rho V^2)/2 \leq p_{max}
\]

Quasi equilibrium glide condition is considered to be the soft constraint. Equilibrium glide refers to aerodynamic lift balancing the gravitational and centrifugal forces as given by (13).

\[
L \cos \sigma = g - \frac{V^2}{r}
\]

where \( \sigma \) is specified bank angle. In equilibrium glide, the flight path angle should be constant, but it usually varies with time. Hence, it is called quasi equilibrium glide condition. In this paper, only heat rate is considered as the hard constraint.
2.2 Terminal constraints

Entry trajectory should achieve the terminal conditions on altitude, velocity, and range-to-go as desired by the requirement of Terminal Area Energy Management (TAEM) phase.

2.3 Constraints on control variables

In this paper, bank angle \( \sigma \) is chosen as the primary control variable. Though angle of attack profile is predefined, following constraints are to be kept in mind while designing it. Considering limitations of the actuators, sudden changes in the control variables are not allowed. Therefore constraints on rates and acceleration rates are given as

\[
\begin{align*}
\alpha_{\text{min}} < \alpha &< \alpha_{\text{max}} \\
\sigma_{\text{min}} < \sigma &< \sigma_{\text{max}} \\
\dot{\alpha} &< \dot{\alpha}_{\text{max}} \\
\dot{\sigma} &< \dot{\sigma}_{\text{max}} \\
\ddot{\alpha} &< \ddot{\alpha}_{\text{max}} \\
\ddot{\sigma} &< \ddot{\sigma}_{\text{max}}
\end{align*}
\]

3. PROBLEM FORMULATION

The entry trajectory optimization problem is to obtain complete control profile i.e. bank angle profile that generates entry trajectories which satisfy the terminal conditions and the path constraints accurately.

3.1 Objective function

The objective function of this paper is considered to be in Mayer form with only terminal constraints as described in (20)

\[
\text{Min } J = |(s(t_f) - s_f^*)| + |(r(t_f) - r_f^*)| + |V(t_f) - V_f^*)| \tag{20}
\]

subject to dynamic model and path constraint (10).

where \( t_f \) is the specified final time of flight. \( s(t_f) \) denotes the range-to-go at final time. The desired values of range-to-go, radial distance, velocity are \( s_f^*, r_f^* \) and \( V_f^* \) respectively.

4. ORTHOGONAL COLLOCATION BASED CONTROL PARAMETRIZATION

The direct methods used for solving trajectory optimization involve discretizing the control and/or state variables so as to convert the trajectory optimization problem to non-linear programming problem (NLP) which is easier to solve.

4.1 Bank angle profile

In this paper, only control variable i.e. bank angle is discretized. A nominal angle of attack value corresponding to different Mach numbers for CAV-H vehicle given by Lu (2014) is considered for simulations. It is to be noted that the bank angle profile should be a smooth continuous function without any sudden variations so as to satisfy (17) and (19). Usually, direct collocation method uses low-order polynomial splines to approximate continuous functions, whereas orthogonal collocation methods are similar but use higher order splines (Kelly, 2017). The bank angle profile is therefore approximated by a high order orthogonal polynomial. This bank angle profile should be discretized at specific points in time called as collocation points which are located at the roots of an orthogonal polynomial. In this paper, collocation points called Chebyshev-Lobatto (CL) points which are located at the roots of Chebyshev polynomial are considered. The roots of Chebyshev polynomial can be calculated easily compared to the roots of Legendre polynomial. The closed form representation of CL interpolation points is given as

\[
\tau_l = \cos(\pi l/N_1), \quad l = 0, 1, ..., N_1 \tag{21}
\]

where \( N_1 \) is the degree of the polynomial and these points lie in the interval \([-1, 1]\). For collocation, the transformation between time domain \( t \in [t_0, t_f] \) to \( \tau \) is required and is done as follows

\[
t = \left( \frac{t_f - t_0}{2} \right) \tau + \left( \frac{t_f + t_0}{2} \right) \tag{22}
\]

The collocation points which are defined by (21) are from 1 to -1. The initial time and the final time has to be mapped to 1 and -1 respectively using (22). The transformed control variable is \( U(\tau), \tau \epsilon [-1, 1] \) and the control variable at the collocation points is expressed as \( U(\tau_l), (l = 0, ..., N_1) \) which is to be found using PIO algorithm. The complete control variable time history is obtained by using Lagrange interpolation polynomials \( \phi_l(\tau) \) given by,

\[
u(t(t)) = \sum_{i=0}^{N_1} \phi_l(\tau)U(\tau_l) \tag{23}
\]

where \( \tau_l, (l = 0, ..., N_1) \) are CGL points and

\[
\phi_l(\tau) = \prod_{m=0, m \neq l}^{N_1} \frac{\tau - \tau_m}{\tau_l - \tau_m}, \quad l = 0, ..., N_1 \tag{24}
\]

The schematic diagram of the algorithm is shown in Fig. 2

5. PIGEON INSPIRED OPTIMIZATION

The PIO algorithm is proposed by Duan and Qiao (2014) and is successfully applied to various research problems. PIO is based on swarm intelligence which mimics the movement of a flock of pigeons that exhibit excellent homing behavior. They navigate using the magnetic field of the Earth, the altitude of the Sun and landmarks. The mechanism of using the magnetic field of the Earth and altitude of the Sun is denoted as map and compass operator and the other operator is called as landmark operator. In the initial phase of their journey pigeons use map and compass operator and later shift to landmark operator as they approach the destination.

5.1 Map and Compass Operator

By sensing the magnetic field of the Earth, pigeons can locate themselves relative to their destination. This ability is termed as map operator. Using the altitude of the Sun, they can adjust their flying direction which is regarded as the compass operator. These two operators are formulated as follows.
The position of the pigeon $i$ at each iteration, an initial set of pigeons are randomly generated in the given search range. The position of the pigeon $i$ is given by (25).

$$X_i = [x_{i1}, x_{i2}, ..., x_{iD}] \text{ where } i = 1, 2, 3...N$$  

(25)

The velocity of the pigeon $i$ is given by (26).

$$V_i = [v_{i1}, v_{i2}, ..., v_{iD}] \text{ where } i = 1, 2, 3...N$$  

(26)

Each position of the pigeon represents a possible solution and returns an objective function value defined by (20). The pigeon with minimum objective function value will lead the other pigeons in a minimization problems. All the pigeons adjust their position and will try to follow the pigeon that corresponds to the best objective function value i.e minimum objective function. By using this logic, the position and velocities of the pigeons are updated in each iteration $k$ as per following update logic

$$V_i(k) = V_i(k-1).e^{-Rk} + rand. (G(k-1) - X_i(k-1))$$  

(27)

$$X_i(k) = X_i(k-1) + V_i(k)$$  

(28)

where $G(k-1)$ is the position of the pigeon corresponding to the best objective function achieved till the current iteration. $rand$ is a random number in $[0,1]$ and $R$ is called as the map and compass operator. The term $V_i(k-1).e^{-Rk}$ in above update equation gives the pigeons former flying direction.

5.2 Landmark operator

As the pigeons approach their destination, they shift to landmark operator. Some of them can identify the landmarks and can fly directly to their destinations. The remaining pigeons will follow them and reach the destination.

The number of iterations that indicates the shift in the operator is denoted as $k_c$. It is chosen to be 75% of the maximum number of iterations given by $k_c = 0.75k_{max}$ according to Duan and Qiao (2014). When the current iteration $k = k_c$, landmark operator is initiated. In this operator, half of the pigeons with positions nearer to the pigeons with minimum objective function $G(k-1)$ are selected. The center of these pigeons is found using (29).

$$X_c(k) = \frac{\sum_{N_p(k)} X_i(k-1).fitness(X_i(k-1))}{N_p(k) \sum_{N_p(k)} fitness(X_i(k-1))}$$  

(29)

where $fitness(X_i(k)) = \frac{1}{\epsilon_{\text{min}}(X_i(k)) + \epsilon}$ for a minimization problem and $N_p$ is the current reduced population as given below

$$N_p(k) = \frac{N_p(k-1)}{2}$$  

(30)

Using the center of pigeons $X_c(k)$, the positions of the pigeons is updated as follows

$$X_i(k) = X_i(k-1) + rand. (X_c(k) - X_i(k-1))$$  

(31)

In this operator, pigeons that are close to the pigeons with minimum objective function are considered as pigeons that are familiar with the landmarks. Pigeons which are not familiar with the landmarks, adjust their positions and follow the center of the pigeons that are familiar with the landmarks. At the end of iterations $k_{max}$, pigeon corresponding to minimum objective function value will be the pigeon with the best position. PIO algorithm as applied to the current problem is presented in the flowchart discussed in Fig. 3. The path constraints can be handled by penalty factor approach described in Sushnigdha and Joshi (2017).

6. LQR TRAJECTORY TRACKING LAW

The bank angle profile that minimizes the objective function is obtained using PIO algorithm. This bank angle profile is used to integrate the equations of motion and the reference trajectory profiles are generated. These trajectories are to be tracked in the presence of entry state dispersions using Linear Quadratic Regulator (LQR) technique. The equations of motion of re-entry vehicle are non-linear that are to be linearized about the reference trajectory and control. The trajectory tracking problem is redefined in terms of deviation variables of state and control given by $\delta x$ and $\delta u$ respectively. The optimal control problem is redefined to find the deviation in control $\delta u$ that minimizes the performance index $J$ as given in (Dukeman, 2002)
where, $A$ is considered as a locally time-invariant system at each time $t$. The weighing matrices with their off-diagonal elements as 0. Hence, linear control law given below is calculated as

$$
\dot{x} = A(t)\delta x + B(t)\delta u
$$

subject to

$$
\delta x = \begin{bmatrix} \sigma_c \\ \alpha_c \end{bmatrix} = \begin{bmatrix} \sigma_{ref} \\ \alpha_{ref} \end{bmatrix} + \frac{\mathbf{R}}{\mathbf{Q}} \mathbf{x}
$$

where $A(t)$ and $B(t)$ are obtained by linearizing the nonlinear equations of motion about the reference trajectory and control. $A(t)$ and $B(t)$ are approximated at discrete times $t_k$ as constants. Therefore, the system defined by (33) is considered as a locally time-invariant system at each time $t_k$. Hence linear control law given below is calculated as given below

$$
\delta u = -K(t_k)\delta x
$$

where the feedback gains $K(t_k)$ are constants at each $t_k$. These gains are obtained by minimizing the steady-state regulator performance criterion:

$$
J = \int_{t_0}^{t_f} (\dot{x}^T(t)Q\delta x(t) + \delta u^T(t)R\delta u(t)) \ dt
$$

where $Q$ is positive semi-definite and $R$ is positive definite weighing matrices with their off-diagonal elements as 0 and they are selected based on Bryson’s rule described in (Dukeman, 2002).

In case of dispersions in the entry conditions, final desired range-to-go $s^*$ should still be achieved. Since range-to-go is a function of $r, V, \gamma$, they are considered as the reference profiles to be tracked. The control variables now are angle of attack and bank angle, with corresponding deviation variables as $\delta \alpha$ and $\delta \sigma$ respectively. $\alpha_{ref}$ corresponds to the nominal angle of attack chosen for generating the reference trajectory. $\sigma_{ref}$ is the bank angle profile corresponding to reference trajectories. $r_{ref}, V_{ref}$ and $\gamma_{ref}$ are the reference trajectory parameters. $r, V, \gamma$ are the current trajectory parameters. The actual guidance commands required to track the reference trajectory parameters are

$$
\begin{bmatrix} \sigma_c \\ \alpha_c \end{bmatrix} = \begin{bmatrix} \sigma_{ref} \\ \alpha_{ref} \end{bmatrix} + \frac{\mathbf{R}}{\mathbf{Q}} \mathbf{x}
$$

The change in guidance commands with respect to the reference values are obtained using equation given below

$$
\begin{bmatrix} \delta \sigma \\ \delta \alpha \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \begin{bmatrix} r - r_{ref} \\ V - V_{ref} \\ \gamma - \gamma_{ref} \end{bmatrix}
$$

The matrices $A$ and $B$ are given below

$$
A = \begin{bmatrix} 0 & g \sin \gamma \\ \frac{H}{a31} & \frac{H}{a32} \\ \frac{a32}{a33} \end{bmatrix} \begin{bmatrix} V \cos \gamma \\ \frac{V}{g} \\ \frac{V}{g} \cos \gamma \end{bmatrix}
$$

$$
B = \begin{bmatrix} 0 \\ -L \sin \sigma \\ -L \cos \sigma \end{bmatrix}
$$

where $H = 6932m$ is the scale height. $A$ and $B$ are to be evaluated at each time instant about the reference trajectory variables and controls. The gains $K$ are obtained using lqr function in MATLAB with inputs to the function as the matrices $A$, $B$, $Q$ and $R$. Matrices $Q$ and $R$ are chosen based on maximum allowable deviation parameters $\delta r, \delta V, \delta \gamma, \delta \sigma, \delta \alpha$ from reference values.

$$
Q_1(\delta r_{max})^2 = Q_2(\delta V_{max})^2 = Q_3(\delta \gamma_{max})^2 = Q_4(\delta \sigma_{max})^2 = Q_5(\delta \alpha_{max})^2
$$

where “max” denotes the maximum tolerable deviations from the reference values for that particular variable.

7. SIMULATION RESULTS

The number of collocation points $N_1$ are chosen to be 11. Therefore, 11th degree Lagrange interpolation polynomial is to be used. The search dimension of unknown variables in PIO is $D = 12$. The bank angle at the collocation points are initialized in the range of $[-89, 89]$ with pigeons population of $N = 30$, the maximum number of iterations in PIO is $k_{max} = 60$ and the map and compass operator $R = 0.2$. Simulation is carried out using only the map and compass operator. The maximum admissible heat rate is
\[ Q_{\text{max}} = 7 \times 10^6 \text{W/m}^2 \]

with the final time \( t_f \) set to be 2030s. The nominal initial entry conditions are taken from Lu (2015) along with dispersions is given in Table 1 and desired terminal conditions are given in Table 2. Figure 4 gives the plot of minimum cost function obtained in each iteration. The final obtained minimum objective function value is 0.0492.

Table 1. Initial entry conditions and corresponding dispersions

<table>
<thead>
<tr>
<th>( r_0 ) (m)</th>
<th>( \theta_0 ) (deg)</th>
<th>( \phi_0 ) (deg)</th>
<th>( V_0 ) (m/s)</th>
<th>( \gamma_0 ) (deg)</th>
<th>( \psi_0 ) (deg)</th>
<th>( s_0 ) (nmi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference trajectory</td>
<td>121520+R_0</td>
<td>72.42</td>
<td>7400</td>
<td>-1</td>
<td>35</td>
<td>6000</td>
</tr>
<tr>
<td>Dispersions</td>
<td>304.8</td>
<td>-0.1</td>
<td>+0.1</td>
<td>30.48</td>
<td>-0.1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. Final conditions to be achieved

<table>
<thead>
<tr>
<th>( h_f ) (m)</th>
<th>( V_f^* ) (m/s)</th>
<th>( s_f^* ) (nmi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28000+R_0</td>
<td>2000</td>
<td>50</td>
</tr>
</tbody>
</table>

Fig. 4. Variation of cost function with iteration

Fig. 5. Bank angle profile

The complete reference control input time history after interpolation is shown in Fig. 5. It is observed bank angle variation is smooth with bank angle rate and its acceleration the capability of the vehicle. Initial bank angle variation till 200s has no influence on vehicles trajectory as the vehicle is at higher altitudes where the density is very small leading to zero lift. Bank angle modulation becomes effective once the vehicle enters the denser atmosphere.

Changes in bank angle are directly linked to rate of change of flight-path angle \( \dot{\gamma} \) and heading angles \( \dot{\psi} \) as given in (5) and (6) respectively. Flight-path angle is oscillating as seen in Fig. 7. Flight-path angle is related to altitude rate as well, due to fluctuations in flight-path angle, altitude profile also exhibits oscillations as shown in Fig. 8. As the altitude profile has oscillations, there is corresponding fluctuations in the heat rate values as well as load factor as seen in Fig. 11 and Fig. 12. Range-to-go and velocity are monotonically decreasing with time as seen in Fig. 10 and Fig. 9 respectively.

The reference profiles of \( r, V, \gamma \) are tracked using lqr function in MATLAB for entry dispersions indicated in Table 1. The requirement of additional bank angle command to track the reference profiles is observed to be less compared to the required angle of attack command as seen in Fig. 5. At high Mach numbers, an additional angle of attack of 3 deg compared to the reference angle of attack \( \alpha_{\text{ref}} \) is demanded and about 0.8 deg at Mach number 14 as seen in Fig. 6 so as to overcome the dispersions and track the desired reference profiles. It is observed that due to the dispersion in entry conditions, there is a deviation of actual trajectories from the reference trajectories in the initial phase of entry. Later the controller has managed to track the reference trajectory effectively as in Figs. 8, 9, 7 and 10. It is observed that the terminal errors are within the tolerance limits and heat rate constraint is met.
The resulted maximum values of load factor and dynamic pressure are within the acceptable limits.

Fig. 9. Velocity profile

Fig. 10. Range-to-go variation with time

Fig. 11. Heat rate variation with time

Fig. 12. Load factor variation with time

8. CONCLUSION

This paper presents an approach to solve orthogonal collocation problem using PIO algorithm. PIO algorithm manages to find the bank angle values at the collocation points effectively and feasible entry trajectory is obtained with acceptable terminal constraint errors. For reasonable initial entry dispersions, LQR controller tracks the reference profiles accurately. However, there is a need to determine entry trajectory for same initial and desired conditions with less number of collocation points.

REFERENCES


