DEACO: Hybrid Ant Colony Optimization with Differential Evolution

Xiangyin Zhang, Haibin Duan, *Senior Member*, *IEEE*, Jiqiang Jin

***Abstract*—Ant Colony Optimization (ACO) algorithm is a novel meta-heuristic algorithm for the approximate solution of combinatorial optimization problems that has been inspired by the foraging behavior of real ant colonies. ACO has strong robustness and easy to combine with other methods in optimization, but it has the shortcomings of stagnation that limits the wide application to the various areas. In this paper, a hybrid ACO with Differential Evolution (DE) algorithm was proposed to overcome the above-mentioned limitations, and this algorithm was named DEACO. Considering the importance of ACO pheromone trail for ants exploring the candidate paths, DE was applied to optimize the pheromone trail in the basic ACO model. In this way, a reasonable pheromone trail between two neighboring cities can be formed, so as to lead the ants to find out the optimum tour. The proposed algorithm is tested with the Traveling Salesman Problem (TSP), and the experimental results demonstrate that the proposed DEACO is a feasible and effective ACO model in solving complex optimization problems.**

I. INTRODUCTION

ANT Colony Optimization (ACO) was firstly put forward by Dorigo M. in the early 1990s[1], and it was designed to simulate the foraging behavior of real ant colonies. While individual ants have few capabilities, a colony can exhibit quite complex behavior, and in which the parallel computation mechanism is adopted. A schematic diagram of the natural processes that the ACO mimic is shown in Fig. 1 and Fig 2.

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Xiangying Zhang is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing 100083, China (86-10-8235-3329; e-mail: zhangxy@asee.buaa.edu.cn).

Haibin Duan is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing 100083, China (e-mail: hbduan@buaa.edu.cn).

Jiqiang Jin is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing 100083, China (e-mail: 4392180@qq.com).

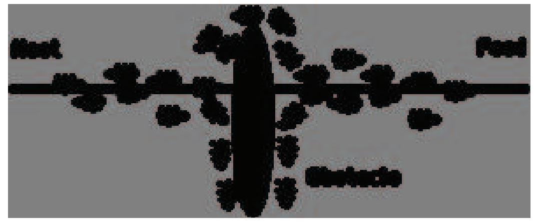

Fig.1 Ant colony are exploring routes with different lengths

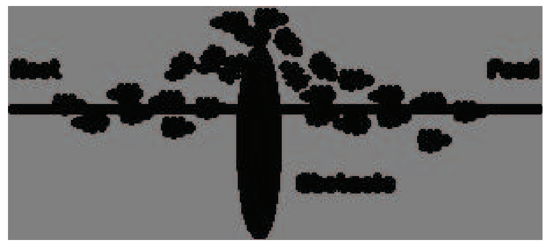

Fig. 2 Schematic diagram of ant colony optimization shows that ant colony has succeeded in finding the shortest route

The principle of the phenomenon in Fig. 1 and Fig 2 is that ants can release some special substance which is named pheromone to the environment while walking [2]. It was found that the ant colony marks a path, communicates information among individuals and decides where to go mainly by the pheromone trail which has relation to the length of the path covered by ants. Moving ants deposits a certain amount of pheromone in the environment, thus making the path by a trail of this substance. At the intersection ants encounter for the first time, they will select one path essentially at random and go forward, meanwhile, they deposit their pheromone determined by the path length. The longer the route ants gained, the smaller the amount of pheromone they deposited. Then, when ants for a second time arrive at the intersection later, each of them prefers in possibility to choose the path richer in pheromone rather than the poorer one. And so, the pheromone trail on the better paths gets stronger and stronger, and ants that choose those paths get more and more, while that of other paths fades away by iteration gradually, and ants choose them get less and less. Thus it generates a positive feedback loop and finally the entire ant colony can converge to the best route. In this process we can learn that it is very important of the pheromone trail for the path exploration of ants.

ACO is a quite novel optimization tactics, which is also a model-based approach for solving hard combinatorial optimization problems. It has been applied extensively to benchmark problems such as the Traveling Salesman

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Problem(TSP), the Job-shop Scheduling Problem(JSP), the Vehicle Routing Problem(VRP), Graph Coloring Problem(GCP), the Quadratic Assignment Problem(QAP) [3]. More recently, the approach has been extended to continuous search domains [4].

In 1995, Storn R and Price K firstly proposed a novel evolutionary algorithm: Differential Evolution (DE) [5, 6]. It is an excellent global optimization algorithm, which is originally proposed as a method for the global continuous optimization. Similar to GA, DE algorithm also contains three basic strategies, namely mutation, crossover, and selection. Firstly, DE randomly generates some initial solutions in the searching space. Then, DE adds the difference vector between two If different members to a third member. In this way, it generates a mutated trial individual. Subsequently, DE combines the mutated trial individual and the original target individual to generate another new individual. If the new individual has a better fitness than the original target one, it will be accepted and replace the original individual in the next generation.

Fig.3 and Fig.4 show the mutation and crossover strategies in DE [5].

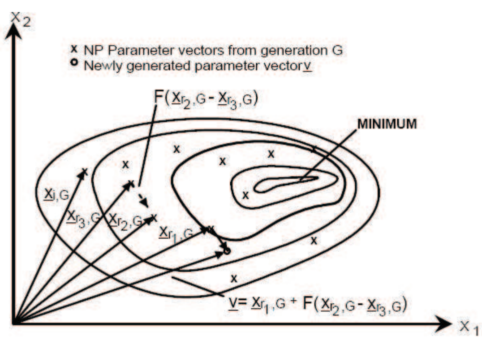

Fig. 3 DE Mutation strategy

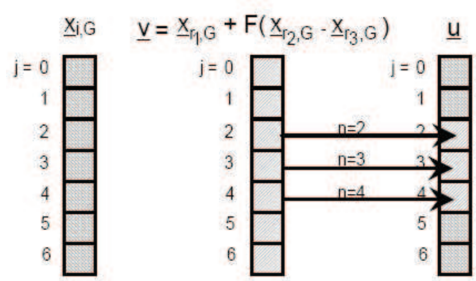

Fig. 4 DE Crossover strategy

Compared with other evolution algorithms, DE algorithm is easier to understand, easy to implement, and it has very strong capabilities in searching the global best solution [8]. Meanwhile, DE has more outstanding performance in parallel computation. For example, DE algorithm is more simple and robust, and it is able to converge to the global optimal solution quickly. It can always find the optimal solution almost during every calculation. Moreover, only a few parameters need setting in DE algorithm, and the same set of parameters can be used for many different problems.

ACO has strong robustness and is easy to combine with other methods in optimization, and the ACO for the heuristic solution of combinational optimization problems enjoys a rapidly growing popularity, but it converges to the optimal solution slowly and has the shortcomings of stagnation that limit the wide application to the various area. While the DE algorithm has a very great ability to search solutions with a fast speed to converge. In order to go step further to optimize the performance of ACO, some exploration on combining ACO and DE may be a good trial. In this paper, we proposed a reasonable combination strategy for the two evolutionary algorithms to generate unexpectedly good results.

The remainders of this paper are organized as follows. The next section introduces the mathematical model of ant colony optimization. Section III proposes a hybrid DEACO algorithm. Then, in Section IV, series of comparison experiments are conducted, which take the example of TSP. Our concluding remarks and future work are contained in the final section.

II. MATHEMATICAL MODEL OF ANT COLONY ALGORITHM

The ant colony optimization mathematical model has first been applied to the TSP [9]. TSP defines the task of finding a tour of minimal total cost given a set of fully connected nodes(cities) and costs associated with each pair of nodes. The tour must be closed and contain each node exactly once. Instances of the TSP come in many different types, such as symmetric (Euclidean or non-Euclidean), asymmetric, dynamic and special TSP [10]. But even within the class of symmetric Euclidean instances, where distance between two cities is taken to be the geometric distance between them, differences can be found.

We define the transition probability from city *i* to city *j* for the *k*-th ant as follows:

$$p_{ij}^{k}(t)=\begin{cases}\dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{k\in allowed_k}[\tau_{ik}(t)]^{\alpha}[\eta_{ik}]^{\beta}} & if \quad j\in allowed_k \\ 0 & otherwise\end{cases} \tag{1}$$

Where $allowed_k$={*N*-tabu$_k$}, and are parameters that control the relative importance of trail versus visibility, is the heuristic desirability, and where is the distance between city *i* and city *j*, is the amount of pheromone trail on edge (*i, j*). After the ants in the algorithm ended their tours, the pheromone trail values of every edge (*i, j*) are updated according to the following formula:

$$\tau_{ij}\left(t+n\right)=\rho\cdot\tau_{ij}(t)+\Delta\tau_{ij} \tag{2}$$

Where ρ is the local pheromone decay parameter, and $\rho\subset(0,1)$. Then, $1-\rho$ represents the evaporation of trail between time *t* and *t*+*n*,

$$\Delta\tau_{ij}=\sum_{k=1}^{m}\Delta\tau_{ij}^{k} \tag{3}$$

Where $\Delta\tau_{ij}^{k}$ is the quantity of per unit length of pheromone trail laid on edge (*i*, *j*) by the *k*-th ant between time *t* and *t*+*n*. In the popular ant-cycle model, it is given by:

$$\Delta\tau_{ij}^{k}=\begin{cases}\frac{Q}{L_k} & if \quad k-th \quad ant \quad uses \quad (i,j) \quad in \quad its \quad tour \\ 0 & otherwise\end{cases} \quad (4)$$

Where Q is a constant and L_k is the tour length of the k-th ant.

This iteration process goes on until a certain termination condition: a certain number of iterations have been achieved, a fixed amount of CPU time has elapsed, or solution quality has been achieved.

III. HYBRID ANT COLONY OPTIMIZATION WITH DIFFERENTIAL EVOLUTION

The ACO pheromone plays a very important role in the path exploration and exploitation. A reasonable distribution of the pheromone trial can directly affect ants to explore their optimal paths. In view of this, we propose a hybrid ACO with DE model, DEACO. We take advantage of DE to make some random deviations disturbance in the pheromone trail of ACO. Thorough this kind of random disturbance, we intend to realize that the pheromone trail between two neighboring cities left by ant colony can reach a more reasonable distribution, which can lead ants to find out the optimal path.

In our proposed DEACO algorithm model, we set the pheromone on the path left by ants in ACO as the object of the Mutation, Crossover and Selection in DE. In solving TSP, the objective function of the pheromone on all sub-paths between two neighboring cities is the length of the best tour found by ants, which is obtained according to the pheromone trail.

Firstly, we should do some slightly adjustment to the ant colony of the basic ACO model. We divide the entire ant colony into several independent ant teams, and the team number is recorded as *Team*, which had better be a restriction of the total ant number *m*. For each ant-team, the amount of pheromone left on the links between each two neighboring cities are recorded as $\tau=\{\tau_i\}$, $i=1,...,Team$. Obviously, τ_i is a $n\times n$ matrix. As to the current pheromone of each ant-team, DE mutation operation takes effect, and the new trial pheromone trail distribution is generated by Equation (5):

$$\tau_{1_i}=\tau_{r_1}+F\times(\tau_{r_2}-\tau_{r_3}), \quad i=1,2,\cdots,Team \quad (5)$$

Where the integers r_1, r_2 and r_3 are chosen randomly from the interval [1, *Team*], that is τ_{r_1}, τ_{r_2}, τ_{r_3} are three pheromone trail individuals, which are selected randomly among all ant-team units, and $r_1\neq r_2\neq r_3\neq i$. F is a real and constant factor between [0, 2], which is named *Constant of Mutation*[5], and it controls the amplification of the differential variation $(\tau_{r_2}-\tau_{r_3})$. Obviously, the smaller the differential variation between two individuals, the weaker the disturbance which it brings about. It signifies that when the pheromone of each ant-team converge to the vicinity of a kind of reasonable pheromone distribution, the disturbance generated through mutation will become weaker automatically.

In our proposed DEACO algorithm, in order to improve the diversification of pheromone trail between cities, we can take advantage of the DE crossover operation to make the new trial pheromone trail τ_{1_i}, which is generated through mutation, combined with the current target pheromone τ_i. DEACO algorithm generates a new pheromone matrix $\tau_{2_i}=\begin{bmatrix}\tau_{2_i}^{1,1} & . & . & \tau_{2_i}^{1,n}\\ . & . & . & .\\ . & . & . & .\\ \tau_{2_i}^{n,1} & . & . & \tau_{2_i}^{n,n}\end{bmatrix}$, $i=1,...,Team$, which can be expressed as follows:

$$\tau_{2_i}^{j,k}=\begin{cases}\tau_i^{j,k}, & if \quad randb\leq CR \quad or \quad rand_k=k,\\ \tau_{1_i}^{j,k}, & if \quad randb>CR \quad or \quad rand_k\neq k,\end{cases} \quad (6)$$

Where, $\tau_i^{j,k}$ denotes the amount of pheromone between city j and k of i-th ant-team, $\tau_{1i}^{j,k}$ denotes the trial pheromone trail between city j and k of the i-th ant-team after the mutation operation, $\tau_{2i}^{j,k}$ denotes the i-th ant-team' pheromone trail between city j and k, after the crossover operation towards $\tau_i^{j,k}$ and $\tau_{1i}^{j,k}$. *randb* is a random positive number between [0, 1]. *CR* is a constant between [0, 1], which is known as *Constant of Crossover*; the larger it is, the greater possibility the crossover operation happens; *CR*=0 represents that no DE crossover occurs. $rand_k$ is a integer number selected randomly between [1,n]; it can pledge that the new generated pheromone matrix τ_{2_i} will surely get at least one element from that mutation trial pheromone τ_{1_i}, otherwise, it is possible that the pheromone trail will not change at all, which can weaken the pheromone exchange between different ant-teams.

In the TSP, ants in each team construct their paths by the transition probability $p_{j,k}$, which can be calculated by their pheromone matrix τ_i. L_best_i denotes the length of the shortest path among all paths obtained by ants, which is the objective function of the pheromone trail τ_i at the same time. Toward the newly generated pheromone trails, and the path explorations of ant colony based on them, should we accept them or not? We need compare the objective function value of both the original target pheromone τ_i and the new τ_{2_i}. After that, we select one solution by so-called *"Greedy"* selection model. If and only if the new pheromone trail individual τ_{2_i} has a better objective function value than the original one, it can be accepted and reserved into the pheromone trail matrix of the next generation; otherwise, the original target pheromone τ_i will remain in the pheromone trail between cities of each ant-team. Thus, we can express the crossover operation to pheromone trail as follows:

$$\tau'_{i,t}=\begin{cases}\tau_{2_{i},t}\,, & if \quad L\_best_{2_i} < L\_best_{0_i}\\ \tau_{i,t}\,, & if \quad L\_best_{2_i} \ge L\_best_{0_i}\end{cases} \quad (7)$$

Where, $\tau_{i,t}$ denotes the original pheromone trail left by the *i*-th ant-team, when the number of iteration is *t*; $\tau_{2i,t}$ denotes, at the *t*-th iteration, the new pheromone trail of the *i*-th ant-team after DE mutation and crossover operation; $\tau'_{i,t}$ is equal to the pheromone matrix which has high objective function value between $\tau_{i,t}$ and $\tau_{2i,t}$. $L\_best_{0_i}$ represents the length of the optimal route gained by $\tau_{i,t}$, which is the objective value of the original pheromone $\tau_{i,t}$; ant-team, while $L\_best_{2_i}$ represents the length of the optimal route gained by $\tau_{2i,t}$, which is also the objective value of the new pheromone $\tau_{2i,t}$ of the *i*-th ant-team.

After the selection operation,the *i*-th ant-team which have generated their tours by the pheromone trail $\tau'_{i,t}$ or $\tau_{2i,t}$, release their own pheromone regarding the length of tours they each covered, and update the selected pheromone trail $\tau'_{i,t}$ to gain the new pheromone trail $\tau_{i,t+1}$. Then pass the new pheromone of each ant-team on to next iteration to continue path exploration and exploitation or stop.

The process of our proposed hybrid DEACO algorithm for solving TSP can be described as follows:

Step 1. Initialization of parameters: set the current number of iteration *Nc*=1; set the maximum number of iteration as *Nc_max*; set the number of ants as *m*, and the number of ant-team as *Team*; set the initial amount of pheromone trail on each link between two cities $\tau^{j,k}=const$, here *const* is a positive constant number; Set other parameters of ACO and DE: *α, β, ρ, Q, F, CR*.

Step 2. Initialization of the ant colony: divide the whole ant colony into different ant-teams, the numbers of ant in each ant-team are recorded in the matrix *T_m*, (1×*Team*); that is, for the *i*-th ant-team, the number of ant individuals is *T_m(i)*; then put ants in each ant-teams on the *n* cities randomly.

Step 3. Set *Nc*=1, begin the first iteration: set *i*=1, the *T_m(i)* ants in the *i*-th ant-team select the city *k* and go forward as the transition probability $p_{j,k}$ calculated by Equation (1), until the whole ant colony finish traveling all the cities in TSP; then update the pheromone trail left on the paths by Equation (2), (3) and (4), to generate $\tau_{i,2}$; set *i*=*i*+1, return to carry out Step 3. until *i*>*Team.*

Step 4. *Nc*=*Nc*+1, take mutation and crossover operation to the original pheromone trail τ_i of each ant-team passed from the former iteration by Equation (5) and (6), and generate the new pheromone trail τ_{2_i}; set *i*=*i*+1, return to Step 4. until *i*>*Team.*

Step 5. Set *i*=1.

Step 6. Ant individuals of the *i*-th ant-team visit all the cities to construct their tours according to the pheromone trail τ_i by the following equation:

$$p_{j,k}=\begin{cases}\dfrac{\left[\tau_i^{\,j,k}\right]^{\alpha}\left[\eta_{j,k}\right]^{\beta}}{\sum_{s\in allowed_{t-m(i)}}\left[\tau_i^{\,j,k}\right]^{\alpha}\left[\eta_{j,k}\right]^{\beta}} & if \quad k\in allowed_{T-m(i)}\\ 0 & otherwise\end{cases} \quad (8)$$

Then calculate length of tours gained by each ants, choose the shortest one, and record it as $L\_best_{0_i}$.

Step 7. Then, each ant individuals of the *i*-th ant-team visit the whole cities in TSP to gain their tours by the pheromone trail τ_{2i} as follow:

$$p_{j,k}=\begin{cases}\dfrac{\left[\tau_{2i}^{\,j,k}\right]^{\alpha}\left[\eta_{j,k}\right]^{\beta}}{\sum_{s\in allowed_{t-m(i)}}\left[\tau_{2i}^{\,j,k}\right]^{\alpha}\left[\eta_{j,k}\right]^{\beta}} & if \quad k\in allowed_{T-m(i)}\\ 0 & otherwise\end{cases}$$

Then calculate length of tours gained by each ants, choose the shortest one, and record it as L_best_{2i}

Step 8. Compare L_best_{0i} and L_best_{2i}, take the DE selection operation by Equation (7), set the τ_i' as τ_i or τ_{2i}.

Step 9. Update the current pheromone τ_i' to gained τ_i of next iteration as follow:

$$\Delta\tau^{j,k}_{T\_m(i)}=\begin{cases}\dfrac{Q}{L_{T-m(i)}}, if \quad T\_m(i)-th \quad antpassed \ (j,k)\\ 0, \quad otherwise\end{cases} \quad (9)$$

$$\Delta\tau_i=\sum_{s=1}^{T\_m(i)}\Delta\tau_s^{j,k} \quad (10)$$

$$\tau_i=\rho\cdot\tau'_i+\Delta\tau_i \quad (11)$$

If select τ_i as τ_i' in Step 8., use the tours gained by ants in Step 6 to update the pheromone; if $\tau_i'=\tau_{2i}$ after selection operation, choose the tours gained in Step 7 to update the pheromone.

Step 10. Set *i*=*i*+1; return to Step 6, until *i*>*Team*.

Step 11. Return to Step 4, until $Nc\ge Nc_{max}$ or other defined termination condition is satisfied.

Step 12. Algorithm stops here and output the best tour and the shortest length.

The above-mentioned flow chart of the DEACO process can also be described in the Fig. 5.

IV. EXPERIMENTAL RESULT

In order to investigate the feasibility and effectiveness of the proposed DEACO, a series of experiments are conducted on Att48TSP (composed of 48 cities) and Berlin52 TSP(composed of 52 cities) using the basic ACO proposed by Dorigo M and the DEACO proposed in this paper.

The DEACO and the basic ACO have been coded in

Matlab language and implemented on PC-compatible with 1024 Mb of RAM under the Windows XP. The parameters of the DEACO and the basic ACO were set to the following values: m=30, $Team$=5, α=2, β=4, ρ=0.3, Q=10, F=2, CR=0.5, τ_{ij}=1, N_{Cmax}=100,.

Fig. 6 shows the best tour gained by basic ACO after 100 iterations. It is obvious that there are cross links in the final result.

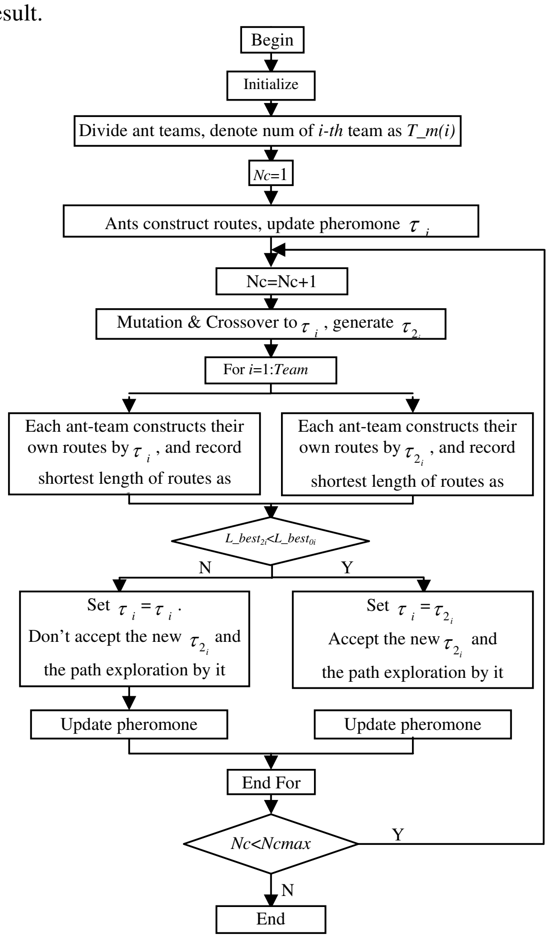

Fig.5 Flow chart of the proposed DEACO

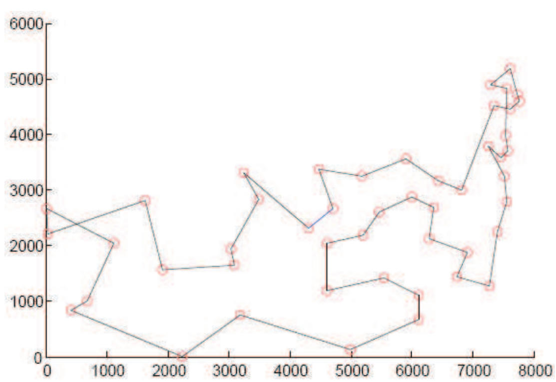

Fig. 6 The best tour of ATT48 by basic ACO

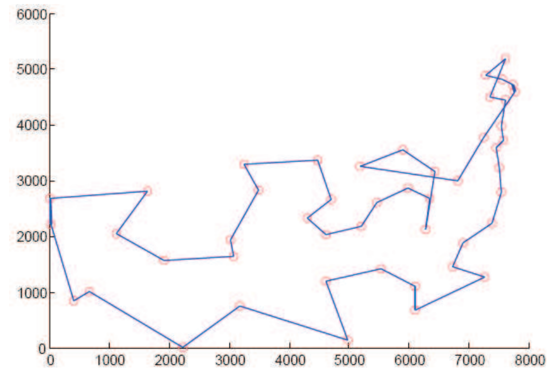

Fig. 7 The best tour of Att48 by DEACO

Fig.7 is the best tour gained after 100 iterations by the DEACO. From Fig 7, it is obvious that there is no crossover path. Fig.8 and Fig.9 show evolution curve of the shortest and average length of the tours gained by ants in each iteration. We can learn that it is much easier for our proposed DEACO to find the optimal tour, and the ants in DEACO algorithm can converge to the optimal tour in the end. Similar results can also be obtained in solving the Berlin52 TSP.

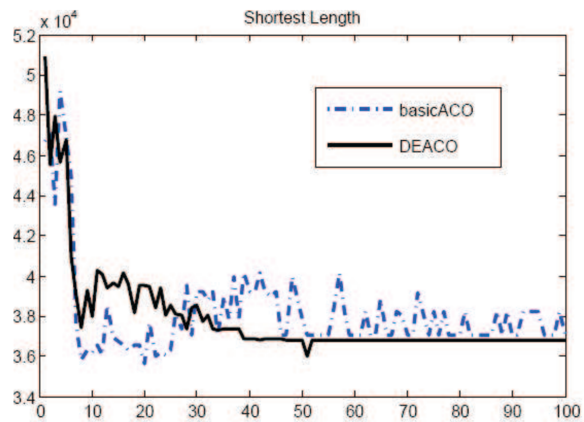

Fig.8 Shortest length comparison between basic ACO and DEACO in solving Att48 TSP

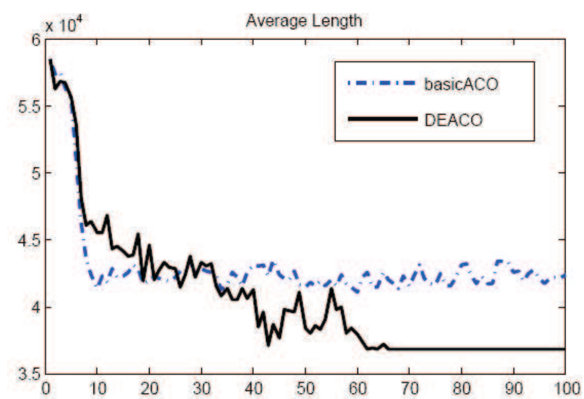

Fig.9 Average length comparison between basic ACO and DEACO in solving Att48 TSP

Fig.10 ~ Fig.13 are experimental results of DEACO solving the Berlin52 TSP.

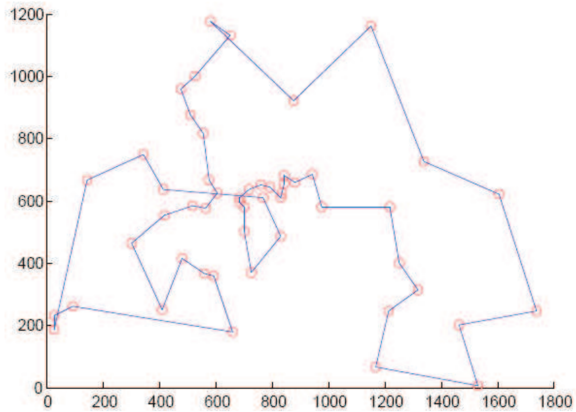

Fig. 10 The best tour of berlin52 found out by basic ACO

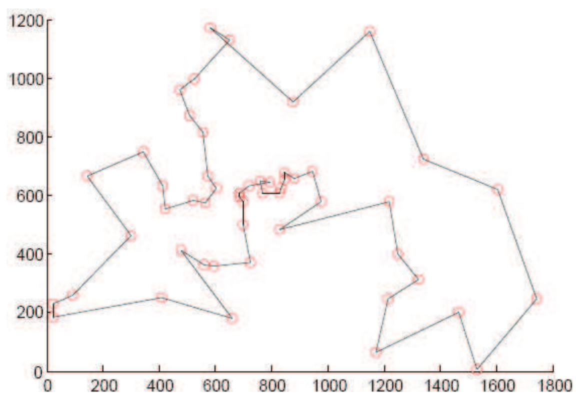

Fig.11 The best tour of Berlin52 found out by DEACO

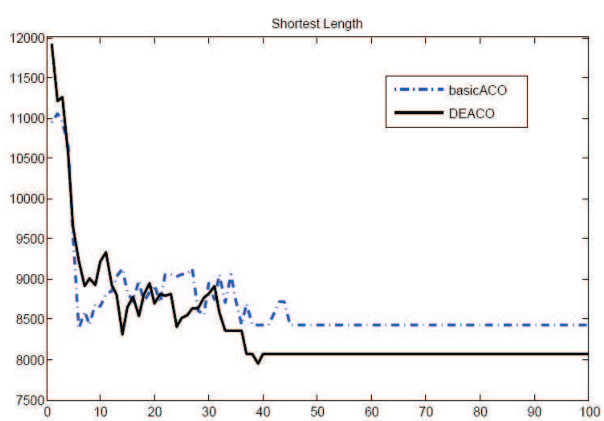

Fig.12 Shortest length comparison between basic ACO and DEACO in solving Berlin52 TSP

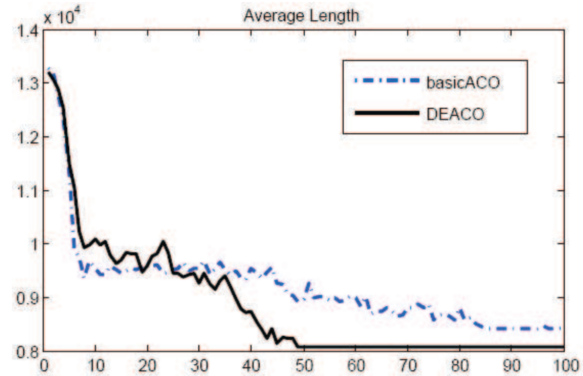

Fig.13 Average length comparison between basic ACO and DEACO in solving Berlin52 TSP

It is obvious that our proposed DEACO algorithm can find better solutions than basic ACO in solving different TSP. Compared with basic ACO. The proposed DEACO has a more excellent performance with strong ability to find optimal solution and quick convergence speed.

V. CONCLUDSIONS AND FUTURE WORK

This paper has presented a hybrid ACO with DE for the TSP. DE is used to optimize the pheromone trail in the basic ACO model. This proposed DEACO is also conducted in Att48TSP and Berlin52 TSP. The series simulation results verify that the proposed hybrid model is a practical and effective algorithm in solving TSP, and also a feasible method for other complex real-world optimization problems.

Our future work will focus on applying the new hybrid approach proposed in this paper to other combinatorial optimization problems, such as JSP, VRP, GCP. Furthermore, we are also interested in seeing how to adjust the numbers of ants and the team in our proposed DEACO.

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