Symbolic Controller with PID Feedback for Locally Linearized System

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Abstract. In this paper, we proposed a novel symbolic controller with PID (proportional integral derivative) feedback for locally linearized system. The state-space of this system is analyzed in Brunovsky coordinates, and then a practical control structure which combines symbolic controller and PID controller is presented. Series of Experiments on an inverted pendulum are conducted, and the results show the feasibility and efficiency of our proposed structure.

Keywords: symbolic controller, PID controller, inverted pendulum, locally linearized system.

1 Introduction

For a wide class of systems, symbolic control was proved to be an effective method in dealing with linear system with feedback encoding. It can reduce the cost of communication and storage resources [1], and solve the kinematic and dynamic constraints simultaneously [2].

Symbolic control is based on the exact discrete-time linear models of control systems. On the contrary to the linear system, it is not possible to obtain the exact models of nonlinear system. Tabuada proposed methodology worked for nonlinear control systems based on the notion of incremental input-to-state stability [3]. However, as to the locally linearized system, the non-linear effects can be treated as a kind of turbulence and be damped by continuous PID feedback control, as the rest of this paper shows.

2 Symbolic Control

Symbolic control is inherently related to the definition of elementary control events, whose combination allows the specification of complex control actions [1]. Letters from the alphabet $\Sigma = \{\sigma 1, \sigma 2, ...\}$ can be utilized to build words of arbitrary length.

A. Bicchi, A. Marigo, and B. Piccoli, introduce a feedback encoding as the Fig.1 shows. In this encoder, an inner continuous (possibly dynamic) feedback loop and an outer discrete-time loop-both embedded on the remote system-are used to achieve richer encoding of transmitted symbols[2].

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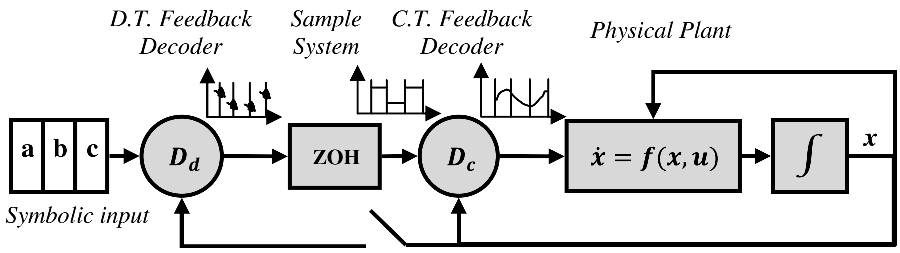


Fig. 1. Nested discrete-time continuous-time feedback encoding

By using nested feedback encoding, all feedback linearized systems are hence additively approachable. The argument can be also directly generalized to multi-input systems by transforming to the Brunovsky form [2].

3 Model of Inverted Pendulum

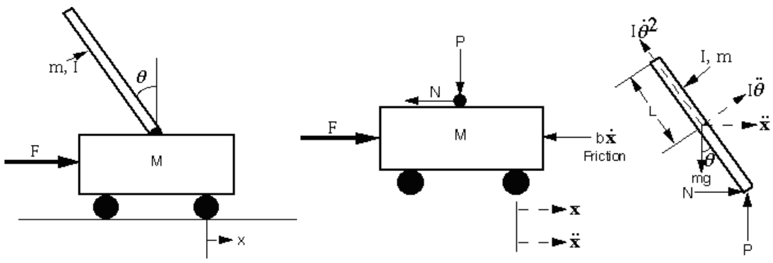


Fig. 2. Inverted pendulum

Consider a nonlinear math model of an inverted pendulum [5]:

$$\frac{d^2x}{dt^2} = \frac{1}{M}\sum F_x = \frac{1}{M}\left(F - N - b\frac{dx}{dt}\right) \tag{1}$$

$$\frac{d^2\theta}{d\theta^2} = \frac{1}{I}\sum \tau = \frac{1}{I}[NLcos(\theta) + PLsin(\theta)] \tag{2}$$

(mass of the cart)	0.5 *kg*
m(mass of the pendulum)	0.2 *kg*
b(friction of the cart)	0.1 *N/m/sec*
l(length to pendulum center of mass)	0.3 *m*
I(inertia of the pendulum)	0.006 *kg*m^2*
F	force applied to the cart
x	cart position coordinate
theta	pendulum vertical angle

By sampling and linearizing the model around its working states, the plant model can be transformed into the state-space form:

$$\begin{bmatrix} x^{(1)} \\ \theta^{(1)} \\ x^{(2)} \\ \theta^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2.67 & -0.18 & 0 \\ 0 & 31.18 & -0.45 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1.82 \\ 4.55 \end{bmatrix} u \tag{3}$$

The *x* position is the controlled variable and *u* denotes the *F*.

4 Symbolic Controller with PID Feedback

4.1 State Space Analyze

If a controller was designed under the control laws mentioned above and applied to the linear model of the cart Eq. (3), the response is identical to the results in Ref. [1] (see Fig. 3(a)).

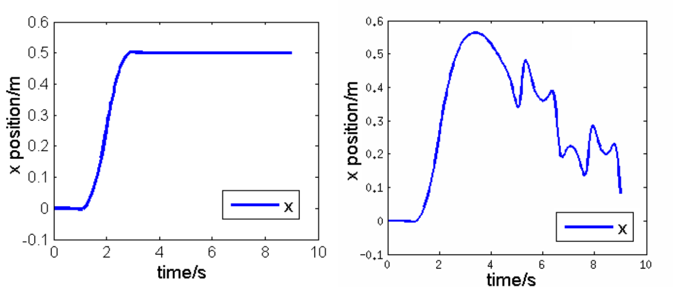


Fig. 3. Responds of the inverted pendulum using symbolic controller on: a)linear model b)nonlinear model

However, if this controller applied to the nonlinear model of the system, the responds of the system is unstable (see Fig. 3(b)), which indicates that the problem should be caused by the nonlinear factor of the plant.

If the states are observed in the continuous time, the result is shown as Fig.4 (a). This shows that in the span of two sampling time, there exists a large range of erratic states in the system states. Since the model is strictly linear, the system arrives at its states according to the sampling time precisely and then updates the new feedback control values with the states according to sampling time. However, the nonlinear factor of the plant caused minor differences against the ideal linear system. Such differences can become larger due to the symbolic feedback controller cannot damp such turbulence, and then the system finally becomes unstable.

A solution to this problem is adding a continuous feedback (PID in this paper) to the system to enhance its robust quality, so the error could be reduced gradually, preventing the feedback of symbolic control being affected.

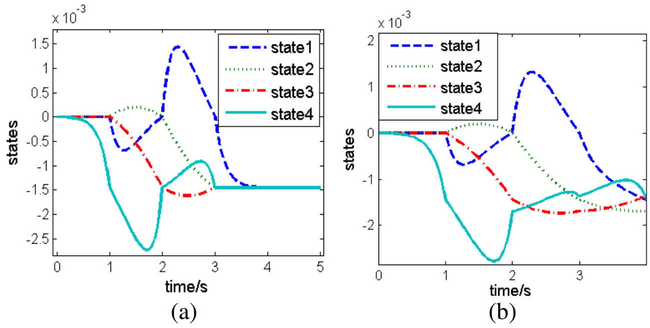


Fig. 4. Brunovskycoordinates states in continuous time of: a) linear model. b)nonlinear model.

4.2 Structure of the Controller

After adding the amend signal of both feedback controller and PID controller, the final system structure is shown with the following Fig. 5.

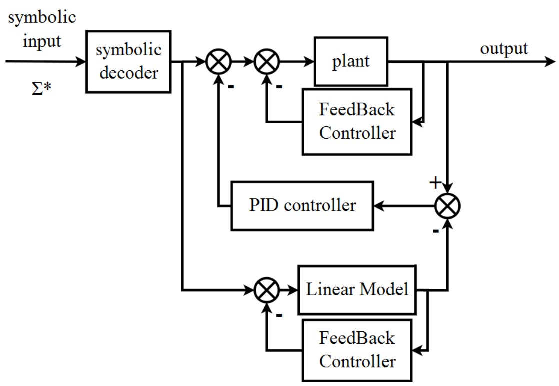


Fig. 5. Structure with symbolic control and PID controller

In this structure, the linear model of the plant is used to generate the output which the symbolic controller is designed for and provide a trace for the output of the plant. Then the difference of the plant and the linear model's outputs is damped by a PID controller to force their behavior to be identical.

5 Experimental Results

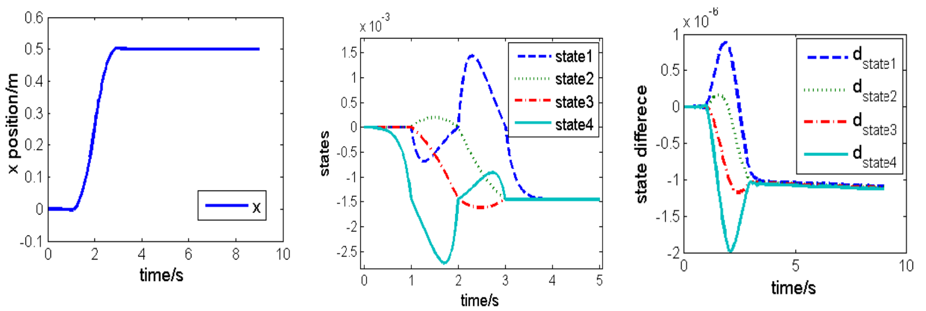
In this section, we will illustrate the power of the PID-symbolic hybrid controller by solving the problem of controlling the nonlinear inverted pendulum.

First of all, by observing the linear model of the pendulum (Eq. (3)) and using the methods in section 2, the equilibrium manifold of the system is gained as $\xi = \{x \in \mathbb{R}^4 | x = (\alpha, 0,0,0)\}$ then a symbolic feedback matrix K and the matrices S, V can be calculated according to the linear state-space equation [1]. In here, $K = [-0.000 \quad 7.0840 \quad -0.11161 \quad 1.2846]^T$. The equilibrium manifold can be written into the Brunovsky coordinates form: $\xi = \{\beta \cdot 1_4\}$, and we assume that the desire precision is 0.01 and the corresponding scale factor$\gamma = 0.0029m$.

Then we specify a PID controller for the system:$K_P = 100, T_I = 1, T_D = 20$ and set the input of the PID controller as the difference of *theta*.

Finally, a symbolic control value$x_t$is given to 0.5, then the symbolic control value is $v_i = x_t/\gamma = 0.5/0.0029 = 1724 \approx 172$($i$=1,2,3,4) and the motion of the cart and the pendulum are shown in Fig. 6 (b).

Compared with the result in Fig. 3(b), the system is stabilized via the PID-symbolic controller (see Fig. 6(a)). Also, under the Brunovsky coordinates (see Fig.6(c)), the figure is similar to the idea linear system states (see Fig. 4(a)) .The static error between two states exits because the input of PID controller is the difference of *theta*, adding an extra PID controller will reduce this error.



(a) (b) (c)

Fig. 6. a) Corresponding Brunovsky coordinates states in continuous time using symbolic-PID controller. b) Corresponding Brunovsky coordinates states in continuous time using symbolic controller. c) Differences between the linear model system states and the nonlinear model states.

Moreover, a larger control value is x_t given to the system to test its ability. A final displacement x_t is set as *15m* and the corresponding symbolic control values are given to the system. The responds of the x and *theta* show in the Fig. 7(a) and Fig. 7(b).

According to the figures above, the pendulum arrives at the final position accurately without overflows in a short time. Moreover, the system withstands a 50 degrees oscillation of *theta* which contains a significant part of nonlinear factors and keeps stable. This result shows that the control structure overcomes the nonlinear factors of the system effectively.

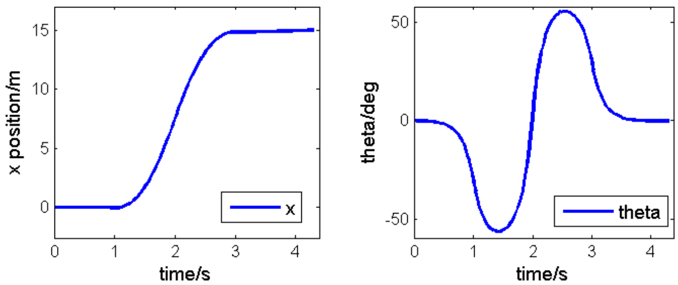


Fig. 7. a) *x* responds for a larger x_t. b): *theta* responds for a larger x_t

6 Conclusion

In this paper, we proposed a structure which combines the continuous PID and symbolic control to control the nonlinear inverted pendulum and obtained stable control results and successfully eliminated the nonlinear factors of the system. However, evaluating the range of applications of this structure on locally linearized systems should be further studied.

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