An Improved Quantum Evolutionary Algorithm with 2-Crossovers

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Abstract. Quantum evolutionary algorithm (QEA) is proposed on the basis of the concept and principles of quantum computing, which is a classical meta-heuristic algorithm for the approximate solution of combinatorial optimization problems that has been inspired by the principles of evaluation of living organisms in nature. QEA has strong robustness and easy to combine with other methods in optimization, but it has the shortcomings of stagnation that limits the wide application to the various areas. In this paper, a hybrid QEA with 2-crossovers was proposed to overcome the above-mentioned limitations. Considering the importance of randomization, 2-crossovers were applied to improve the convergence quality in the basic QEA model. In this way, the new-born individual after each updating can to help the population jump out of premature convergence. The proposed algorithm is tested with the Benchmark optimization problem, and the experimental results demonstrate that the proposed QEA is a feasible and effective in solving complex optimization problems.

Keywords: Quantum evolutionary algorithm (QEA); Genetic algorithm (GA); Qubit chromosome; Crossover; premature.

1 Introduction

Since the concept of quantum was put forward, there was a revolution coming in the field of computing, and it was coming from quantum-the smallest of all places: the subatomic particles that form the basis of all matters.

Quantum computing has promised prodigious powers in the past years. Its basic currency, the qubit, exists in an ON or OFF verge, which you will never know until it's read out. Therefore, if you could operate on K qubits, a potentially vast space of 2^K values opens up for computation which means that we can solve many computing problems at the same time, which saves you a lot of time. The fundamental operation on qubits is a rotation. We have logic gates to combine the rotations. The algorithm is based on these logic gates. In principle, these algorithms can perform calculations far beyond classical computation's conceivable reach.

Genetic algorithm (GA) was firstly put forward by J. Holland in 1970s to study the self adaptation behavior of natural system [1]. It's a classical meta-heuristic algorithm

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for the approximate solution of combinatorial optimization problems that has been inspired by the principles of evaluation of living organisms in nature. The application of GA needs no initiating knowledge of the system, and it isn't limited by the form and property of the problem. Guided by fitness function and principle of probability, it can search in global according to self adaptation by using selection, crossover and mutation. Therefore, it's a comprehensive optimization method with extensive application in terms of processing complex non-linear problems.

GA has strong robustness and is easy to combine with other methods in optimization, but it has limited population size and the problems of premature convergence and stagnation that limit the wide application to the various area often exist. Qubit chromosomes enjoy a rapidly growing population and strong randomization.

To overcome the above-mentioned shortcomings of GA, quantum evolutionary algorithm (QEA) is proposed on the basis of the concept and principles of quantum computing. In QEA, qubit chromosomes, which can represent a linear superposition of solutions, are adopted to maintain solution diversity and overcome premature convergence. At the same time, quantum rotation gate, which make full use of the information of the current best individual, is used to update individual and avoid stagnation [2].

The common QEA uses qubit gate rotation in mutation and whole interference in crossover [3]. By using rotation operation, we can make full use of the information of the currently best individual to perform the next searching process, and the whole interference can avoid prematurity. In this way, the global search capacity can be greatly improved, while the convergence speed is slowed down. In order to further improve the whole performance of QEA, a new hybrid strategy was proposed in this paper.

The remainder of this paper is organized as follows. The next section introduces the main process of common QEA. Section 3 proposes a hybrid QEA model with 2-crossovers. Then, in Section 4, series of comparison experiments are conducted. Our concluding remarks and future work are contained in the final section.

2 Basic QEA

2.1 Qubit Chromosome

In QEA, a qubit chromosome as a string of *n* qubits can be defined as follows [4]:

$$q = \left[\begin{array}{c|c|c|c} \alpha_1 & \alpha_2 & \cdots & \alpha_m \\ \beta_1 & \beta_2 & \cdots & \beta_m \end{array}\right] \tag{1}$$

where $|\alpha_i|^2+|\beta_i|^2=1$, $i=1,\ldots, m$, and m is the number of qubits and also the string length of the qubit individual. $|\alpha_i|^2$ gives the probability that the qubit will be found in the state of '0' and $|\beta_i|^2$ gives the probability that the qubit will be found in the '1' state. A qubit chromosome is able to represent a linear superposition of all possible solutions. It has a better characteristic of diversity than classical chromosome[5]. The process to get classical chromosome is: bring a random number between 0 and 1, if it's bigger than $|\alpha_i|^2$, this bit in classical chromosome is '1', else '0' is chosen.

2.2 Quantum Mutation

The standard mutation operation is totally random without any directions, so the speed of convergence is slowed down. But in QEA, the qubit representation can be used as a mutation operator. Directed by the current best individual, quantum mutation is completed through the quantum rotation gate U(θ),then the $[\alpha_i \ \ \beta_i]^T$ is updated as:

$$\begin{bmatrix} \alpha'_i \\ \beta'_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \tag{2}$$

Look up the Table 1 to find out the right θ_i, which is determined by both quantum and classical chromosome.

Table 1. Rotation angle

x_i	$best_i$	$f(x)>f(\text{best})$	θ_i			
			$\alpha_i\beta_i>0$	$\alpha_i\beta_i>0$	$\alpha_i\beta_i>0$	$\alpha_i\beta_i>0$
0	0	*False*	0	0	0	0
0	0	*True*	0	0	0	0
0	1	*False*	0	0	0	0
0	1	*True*	-0.05π	0.05π	±0.05π	0
1	0	*False*	-0.05π	0.05π	±0.05π	0
1	0	*True*	0.05π	-0.05π	0	±0.05π
1	1	*False*	0.05π	-0.05π	0	±0.05π
1	1	*True*	0.05π	-0.05π	0	±0.05π

x_i is the i-th bit of the current classical chromosome, $best_i$ is the ith bit of the current best classical chromosome, $f(x)$ is the adaptation function [6].

The Figure 1 below describes the polar plot of the rotation operation on qubit. It tells us the reason why the rotation gate can increase the speed of convergence obviously [7].

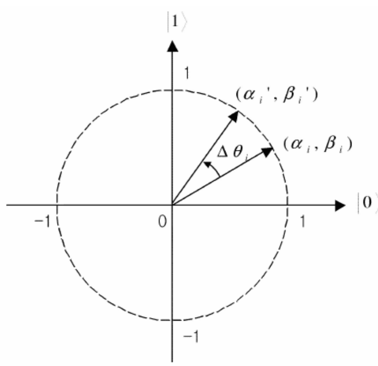

Fig. 1. Polar plot of the rotation gate for qubit chromosome

2.3 Quantum Whole Interference Crossover

This kind of crossover operation is constructed by the interference characteristic of qubit. All the quantum chromosomes are involved in. For example, when the population number is 5 and the length of chromosome is 6, the table 2 below introduces a kind of operation:

Table 2. The whole interference crossover operation

1	A(1)	E(2)	D(3)	C(4)	B(5)	A(6)	E(7)
2	B(1)	A(2)	E(3)	D(4)	C(5)	B(6)	A(7)
3	C(1)	B(2)	A(3)	E(4)	D(5)	C(6)	B(7)
4	D(1)	C(2)	B(3)	A(4)	E(5)	D(6)	C(7)
5	E(1)	D(2)	C(3)	B(4)	A(5)	E(6)	D(7)

The whole interference crossover operation can make full use of the information in the chromosome, improve the unilateralism of classical crossover and avoid premature convergence and stagnation problems.

3 The Proposed Hybrid QEA with 2-Crossovers

The QEA provide new ideas to improve the traditional GA. Firstly, the information in a quantum chromosome is more than that in a classical chromosome, the number of population is decreased and the diversity is improved. Secondly, the mutation operation is no longer totally random but directed by some rules to make the next generation better and increase the speed of convergence. Thirdly, whole interference crossover operation can avoid premature convergence and stagnation problems.

When the whole interference crossover operation try to produce new solution to avoid premature convergence, it's not good for the maintaining of current good solutions. Then, the convergence speed of QEA is decreased. In order to improve the convergence speed and avoid premature convergence, we proposed a hybrid QEA. The result is extraordinary both in theory and experiments.

In our proposed hybrid QEA, we use 2-crossover operations, and some improvements are also conducted in the quantum mutation.

The first crossover operation is the classical single point crossover. Roulette selection operation is used to choose two quantum chromosomes from the parent generations, then the child generation is produced by crossover. After this process, two better individuals can be chosen into the next generation by evaluating their fitness. This operation is mainly to improve the convergence speed and preserve the instructive information. Usually, we choose 0.6 to 0.9 as the crossover probabilities in this process.

Then, we evaluate the fitness of the whole population. We choose the best one as the mutation director, because the evaluation is just before mutation operation, the director individual is considered better than anyone else. Therefore, when we use the rules shown in Table 1, $f(x)$ is always smaller than $f(best)$. Here, we only need half of the information presented in Table 1. The table is simplified, thus the evolution process is faster and easier. We choose 0.01 to 0.2 as the mutation probabilities. Although it's not traditional mutation and the individual can also converge by this operation, the mutation probability can't be very high. Because in every generation, we only choose one best individual, and we use quantum chromosome, the selection is full of randomicity, so we're not sure the "best solution" we choose is really the best result. Furthermore, too much probability isn't good for evolution.

Another crossover operation, the whole interference crossover operation, is adopted to prevent premature convergence. It can bring new individual to help the population jump out of premature convergence. We also choose 0.01 to 0.2 as the crossover probability. For it's just used to avoid premature convergence, we also can't use it in high frequency.

The process of our proposed hybrid QEA with 2-crossovers for solving complex optimization problems can be described as follows:

Step 1: Initialization of parameters: bring a random angle ω between 0 and 2*π, α=cos(ω),β=sin(ω), then a qubit is produced. Set other parameters: classical crossover probability-*Pcc*, mutation probability-*Pm*, whole interference crossover probability-*Pic*, the max circulation generation-*germax*, the number of population-*n*, and the length of chromosome-*L*.

Step 2: Produce a classical population by using this quantum chromosomes. It's the original classical population. Evaluate the fitness of each chromosome.

Step 3: Use roulette operation to select parents quantum chromosome, operate crossover in the classical crossover probability. Update the quantum population. Then produce a classical population, and find out the best solution which will be used in the mutation operation.

Step 4: Operate mutation in the mutation probability and update the quantum population. Then produce a new classical population, evaluate the fitness of each chromosome, compare with the old ones and update the classical population.

Step 5: Operate the whole interference crossover in it's probability and update the quantum population.

Step 6: If the stopping criterion is satisfied, the proposed QEA algorithm stops, and output the best solution, else return to Step 3.

The above-mentioned procedures of the proposed hybrid QEA process can also be described in the Figure 2.

Table 3. The concised rotation angle

x_i	$best_i$	$f(x)>f(best)$	θ_i			
			$\alpha_i\beta_i>0$	$\alpha_i\beta_i>0$	$\alpha_i\beta_i>0$	$\alpha_i\beta_i>0$
0	0	False	0	0	0	0
0	1	False	0	0	0	0
1	0	True	0.05π	-0.05π	0	±0.05π
1	1	False	0.05π	-0.05π	0	±0.05π

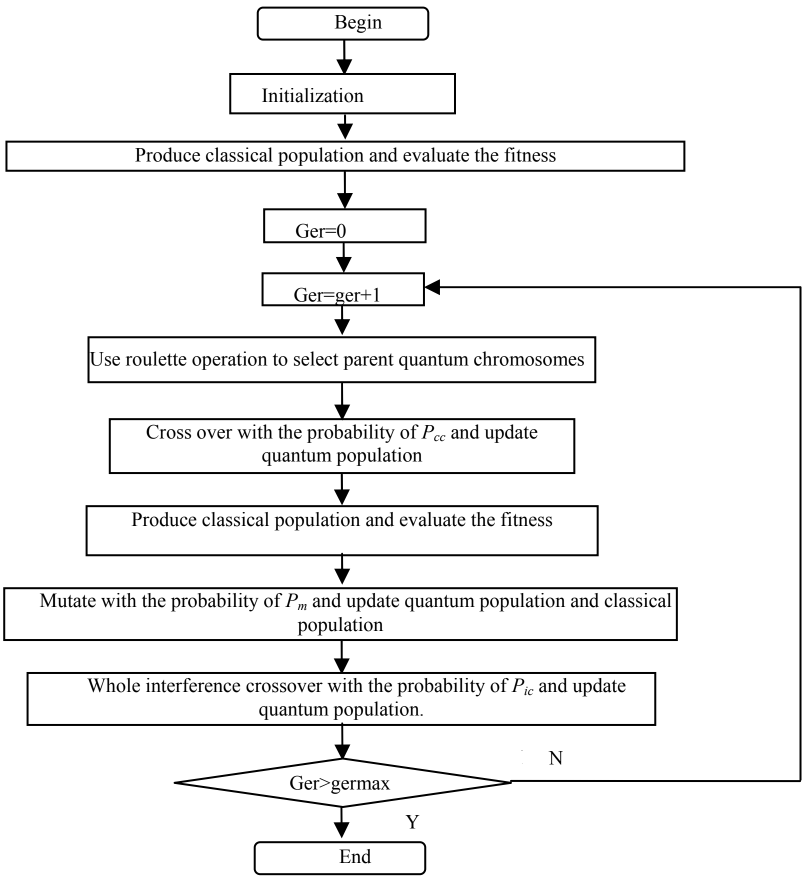

Fig. 2. The proposed hybrid QEA with 2-crossovers

4 Experimental Results

In order to investigate the feasibility and effectiveness of the proposed hybrid QEA with 2-crossovers, a series of experiments are conducted on the Benchmark problem: *Maxf*(*x*)='*x*+10*sin(*x*.*5)+7*cos(*x*.*4)' to find the maximum value. In the three conducted experiments, the differences lie in the adopted crossover operation. The first experiment adopts classical crossover, the second experiment uses whole interference crossover, while the third experiment combines both of them.

The three QEAs have been encoded in Matlab language and implemented on PC-compatible with 1024 Mb of RAM under the Windows XP. The parameters were set to the following values: n=50, L=22, Pcc=0.9, Pic=0.2, Pm=0.2, *germax* is different in the three experiments: 200, 1000and 100.

The original chromosomes is produced with strong randomization. Figure 3 shows the final position of the chromosomes in the first experiment. It's obvious that it's easy to get into premature convergence. The evolution curves presented in Figure 4 also reflects the slow convergence speed.

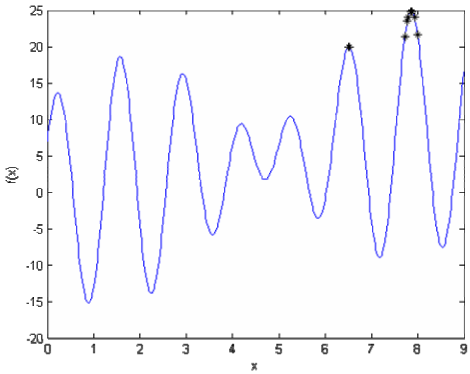

Fig. 3. Final position of the chromosomes in experiment 1

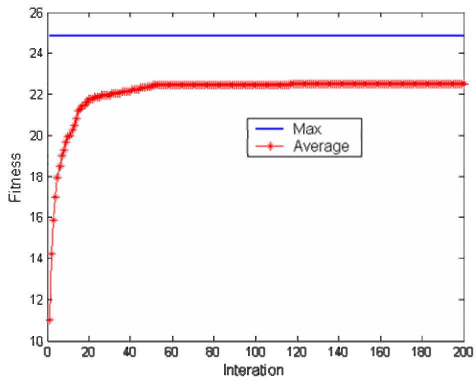

Fig. 4. evolution curve of experiment 1

Figure 5 – Figure 8 show the results in experiment 2 and 3.

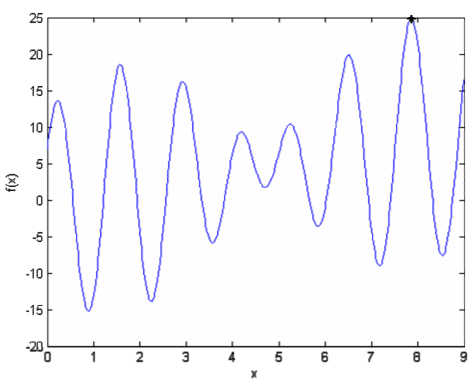

Fig. 5. Final position of the chromosomes in experiment 2

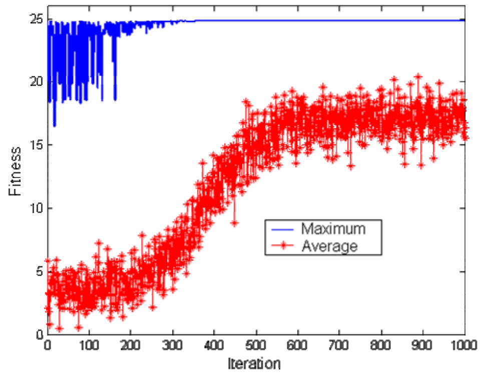

Fig. 6. Evolution curve of experiment 2

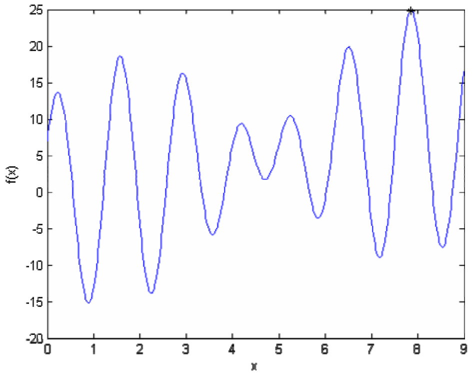

Fig. 7. Final position of the chromosomes in experiment 3

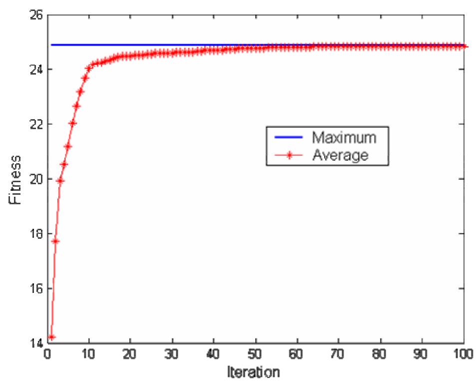

Fig. 8. Evolution curve of experiment 3

It is obvious that our proposed QEA model can find better solutions than the other basic QEAs in solving continuous optimization problems, and the improved QEA model can avoid premature convergence which happens in the first and the second experiments. Our proposed improved QEA with 2-crossovers has a more excellent performance with strong ability to find optimal solution and quick convergence speed.

5 Conclusions and Future Work

This paper has presented an improved QEA with 2-crossovers for solving the continuous optimization problems. The series experimental results verify that the proposed hybrid QEA model is a practical and effective algorithm in solving complex optimization problems, and also a feasible method for other complex real-world optimization problems.

Our future work will focus on applying the newly proposed QEA approach in this paper to other combinatorial optimization problems. Furthermore, we are also interested in the theoretical analysis on the proposed QEA model.

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