Dynamic Multi-UAVs Formation Reconfiguration Based on Hybrid Diversity-PSO and Time Optimal Control

Chen Bai, Haibin Duan, *Senior Member, IEEE*, Chun Li, and Yunpeng Zhang

***Abstract*—This paper proposed a novel hybrid diversity particle swarm optimization (PSO) and time optimal control approach to multiple unmanned aerial vehicles (multi-UAVs) formation reconfiguration in dynamic and complicated environments. Based on the modeling of the dynamic environments, a novel diversity PSO model is proposed. In order to track the dynamic changes, the fitness function changes along with iteration. The procedure for solving the multi-UAVs formation reconfiguration problem is also given in detail. Series simulation results verified the feasibility and effectiveness of the proposed method for solving the multi-UAVs formation reconfiguration problems in complicated and dynamic environments.**

I. INTRODUCTION

IN recent years, multiple Unmanned Aerial Vehicles (multi-UAVs) formation reconfiguration has become a hot topic for the reason that it can be widely applied in both military and civilian fields. When one of the UAVs is damaged or out of control, the formation should be reconfigured to conduct the mission [1]. During the interim of multi-UAVs formation reconfiguration, many constraints should be considered, such as time, threats, collision avoiding, safe distance ensuring, and so on. An appropriate reconfiguration can bring not only high efficiency, but also high accuracy of the mission. The multi-UAVs formation reconfiguration mainly focus on determining a nominal input trajectory for each UAV so that the multi-UAVs group can start from the initial configuration and reach its final configuration at the certain time while satisfying the set of constraints [2].

There have been increasing interests in using evolutionary algorithms in dealing with dynamic optimization problems. Particle Swarm Optimization (PSO) has been firstly used to solve complex optimization problems in 1995 [3], which is a cooperative evolutionary technique developed by Kennedy and Eberhart inspired by the collective individuals' behaviors. These individuals can be regarded as agents, which have the capacity to make their own decision by imitating the successful behavior of their neighborhood while bringing their personal experience there. Once the search process begins, each particle will fly towards the potential solution area according to its own and the other companions' experiments. Owning to some attractive features such as simple model, few parameters, and easy implementation, PSO has been applied in many areas successfully. Therefore, PSO algorithm can be used to control the reconfiguration optimizer to get the desired formation geometry. However, as the multi-UAVs flying environment is dynamic and complicated. When using PSO algorithm for reconfiguration, we must consider the changes of the fitness function and the multi-UAVs flight model.

Generally, most of real-world dynamic optimization problems are often highly non-linear and more complicated[4]. Multi-UAVs formation reconfiguration is also a complicated problem, which can hardly be solved by traditional methods. A basic PSO approach for solving the multi-UAVs formation reconfiguration has been firstly proposed in [8], but this proposed approach is very easy to fall into local optimal result, the convergence speed is also rather slow, and the presented work failed to consider the dynamic model for the multi-UAVs formation reconfiguration. In this paper, a novel hybrid diversity PSO and time optimal control method is adopted to solve the multi-UAVs formation reconfiguration in dynamic environments, which can overcome the shortcomings in [8].

The remainder of this paper is organized as follows. The next section proposes discussion about dynamic factors in complicated environment. Section III introduces the diversity-PSO. Then, in Section IV, the Mathematical Model of Formation Reconfiguration is introduced. Section V proposes the hybrid diversity PSO and time optimal control approach for UAV formation reconfiguration in dynamic and complicated environment. Then, in Section VI, series of comparison experiments are conducted. Our concluding remarks and future work are contained in the final section.

II. MATHEMATIC MODEL OF DYNAMIC ENVIRONMENTS

First of all, an accurate mathematic model of dynamic environments should be built.

One of the common dynamic factors is threat sources. There are two kinds of threat sources: artificial threats and

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Chen Bai is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China (86-10-8233-4107; e-mail: buaabaichen@126.com).

Haibin Duan is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China (e-mail: hbduan@buaa.edu.cn).

Chun Li is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China (e-mail: li_chunchun@126.com).

Yunpeng Zhang is with the School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China (e-mail: zypkkkwz@sina.com)

natural threats. The former include the enemy's radar, battle plane, and artillery and so on. We can choose appropriate models of them under different circumstances. In this paper, we use the circle model to describe these threat sources, and the radius of the circle is the range of threat source. For further research, an appropriate probability density function can be used for describing threat sources more exactly.

The natural threats include landform, wind, and thunder, and so on. These factors are often complicated. We can use the threat analysis tools to build accurate models [5]. Along with the formation reconfiguration, the positions or other factors of threat sources change. And the constraints that we should consider also change. But continuous change is too difficult to be described in a mathematic model. Therefore we use discrete changes in this paper. The values of the fitness function are also discrete. By setting small enough time intervals, discrete mathematic model is fit for formation reconfiguration.

Besides, the nonlinear and variable model of UAV cannot be neglected. Considering the airflow in the real environment between two UAVs, it will enhance the possibility and accuracy of reconfiguration. There should be accurate mathematic models to demonstrate these changeable constraints, which depends on certain circumstances.

In the next two sections, a hybrid of diversity-PSO for dynamic environment and optimal control for reconfiguration is used. We add the changeable constraints to the optimal control. Provided by the optimal control method, the fitness function of the diversity-PSO is dynamic.

III. Diversity-PSO Algorithm in Dynamic Environment

The particle swarm optimization (PSO) algorithm generates populations using stochastic optimization technique. Particles, which are members in the population, have their own positions and velocities, and they fly around the problem space in the swarms searching for the position of optima. In one of the standard versions of PSO algorithm, the velocity and position of i-th particle in dimension d is updated at each time step, according to the following equations[3].

$$v_{id}^{(t+1)} = w \times v_{id}^{(t)} + c_1 r_1 (p_{id}^{(t)} - x_{id}^{(t)}) + c_2 r_2 (p_{gd}^{(t)} - x_{id}^{(t)}) \tag{1}$$

$$x_{id}^{(t+1)} = x_{id}^{(t)} + v_{id}^{(t+1)} \tag{2}$$

Where V_{id} denotes the particle velocity in d-dimension, X_{id} denotes the current particle position (solution) in d-dimension, w denotes the inertia weight. P_i is defined as the best solution (fitness) a particle has achieved so far and P_g is defined as the best value obtained so far by any particle in the population. *rand*( ) is a random function in the range [0,1]. c_1 and c_2 are learning factors, usually $c_1 = c_2 = 2$. If the velocity is higher than a certain limit, called *Vmax*, this limit will be used as the new velocity for this particle in this dimension, thus keeping the particles within the search space.

But the formation reconfiguration requires optimization algorithms not only to find the optimal solution in a short time but also track the optimal solution in a dynamic environment. As the environments change, the fitness isn't static any longer. Therefore tracking the optimal solution along with the change of fitness value is very important in reducing the influence of the out-of-date memory due to the changing environment. The diversity-PSO algorithm is suitable solution to problems mentioned above.

This algorithm includes two steps: detection and response to dynamic environment.

To detect the changes of environment, the information of the relative position and fitness value should be known at first. We use the formulas as follows [6]:

$$P_i(t) = \sqrt{\sum_{d=1}^{n} ((x_{id}(t)) - x_{id}(t-1))^2} \tag{3}$$

$$G_i(t) = \frac{f(X_i(t)) - f(X_i(t-1))}{P_i(t)} \tag{4}$$

$$M_i(t) = |G_i(t) - G_i(t-1)| \tag{5}$$

Where $P_i(t)$ denotes the distance between the previous particle and the later particle, $G_i(t)$ denotes the rate of the fitness along with $P_i(t)$.

Before the changes of environment, $M_i(t)$,which is ratio of the fitness of previous and later generations, is close to 0. When environment changes, $M_i(t)$ will have a jump. But we cannot yet completely ensure that the environment has changed. The values of the best solution in t generation and t-1generation respectively should be compared, if the value changes a lot and $M_i(t)$ has a jump, the detecting process should move on.

The next step is to response the dynamic environment. The dynamic factors cause the change of search space.

Use the diversity function [6]:

$$diversity(s) = \frac{1}{|S| \times |L|} \sum_{i=1}^{|S|} \sqrt{\sum_{j=1}^{N} (p_{ij} - \bar{p}_j)^2} \tag{6}$$

Where S is the number of particles, L stands for the radius of the search space, N is the dimension of the problem. It can describe the space distribution of particles. After analyzing the distribution, whether to reset the particles is determined. If the value of this function is excessively small, it can be ensured that the particles are centered on one certain position and the best solution after change of the environment cannot be achieved.

To reset the particles, we should use modified formulas [6]:

$$v_{id}^{(t+1)} = w \times v_{id}^{(t)} - c_1 r_1 (p_{id}^{(t)} - x_{id}^{(t)}) - c_2 r_2 (p_{gd}^{(t)} - x_{id}^{(t)}) \tag{7}$$

$$x_{id}^{(t+1)} = x_{id}^{(t)} + v_{id}^{(t+1)} \tag{8}$$

These formulas can make particles escape in the opposite direction. Therefore those particle centered on certain position are broken up.

The PSO algorithm provides the optimal solution of input signals.

IV. THE MATHEMATIC MODEL OF FORMATION RECONFIGURATION

In this section we consider N UAVs flying in a horizontal. The control action is from a time $t = 0$ to a terminal time $t=T$.

The equations of motion for one UAV are [7]:

$$\begin{cases} \dot{y}(t) = v(t) \times \cos(\theta) \\ \dot{z}(t) = v(t) \times \sin(\theta) \\ \dot{\theta}(t) = \dfrac{v(t)}{l} \tan \gamma(t) \end{cases} \tag{9}$$

Where y and z are the coordinates of the centroids and θ is the orientation of the body, and l is the length of the body of UAV.

We use the Control Parameterization and Time Discretization (CPTD) method to solve the control problem of the multi-UAVs[5]. We partition the terminal time T into $n_p \in \{1, 2, \cdots\}$ time intervals, $\Delta t_p \in R^+$.

Each of the continuous control input for the i-th UAV can be approximated by a piecewise function with constants.

Considering all the constraints [8], we define the payoff function as:

$$\begin{aligned} J = {} & n_p \times \Delta t_p + M^* \times \hat{g}(\Omega, \Delta t) + \\ & \sum_{i=1}^{N-1} \{ \sum_{j=i+1}^{N} [M_{ij} \times \max(0, D_{safe} - d^{i,j}(x_i(t), x_j(t))) \\ & + M_{ij}' \times \max(0, d^{i,j}(x_i(t), x_j(t)) - D_{comm})] \\ & + M_i'' \times \max(0, R_{threat} - d^i(x_i(t), x_{threat}))\} \end{aligned} \tag{10}$$

Where $\hat{g}(\Omega, \Delta t)$ denotes the terminal constraint. $d^{i,j}(x_i(t), x_j(t))$ denotes the distance between the i-th and j-th UAV. D_{safe} denotes the safe-ensuring distance between two UAVs and D_{comn} denotes the communication-ensuring distance. X_{threat} denotes the position of threat source. M^* denotes the penalty factor of the terminal constraint. M_{ij} denotes the penalty factor of the safe-distance constraint. M_{ij}' denotes the penalty factor of the communication constraint. M_{ij}'' denotes the penalty factor of the threat sources.

In this way, we can use this formula as the value of fitness of the particles.

V. THE PROPOSED METHOD USING DIVERSITY-PSO AND TIME OPTIMA CONTROL ON MULTI-UAVS FORMATION RECONFIGURATION

Diversity-PSO for dynamic environment and optimal control for reconfiguration is used. We give a diamond formation of five UAVs as the final formation after reconfiguration.

Several threat sources, whose positions are changeable, are set to affect the UAV's trochoid. If the positions failed to change with the iterations, the environment cannot be considered as dynamic. Therefore using changeable threat sources is a simple and reasonable method to demonstrate dynamic influence on the reconfiguration.

There are many small circles in the trochoid based on PSO [8]. This phenomenon is caused by partial convergence and the global optimal solution can't be solved.

In the next experiments, we replace (2) with the equation as:

$$x_{id}^{(t+1)} = x_{id}^{(t)} + (rand_1() + k) v_{id}^{(t+1)} + 10^{-6} rand_2() \tag{11}$$

This amelioration can make particles far from the partial optimal solution, and the trochoid is more logical and easier to be applied.

Step 1. Initialize the parameters of multi-UAVs.

Step 2. Initialize the parameters for the proposed diversity-PSO.

Step 3. Consider the given constraints and calculate the fitness of formation reconfiguration used in PSO by formula (10). The unchangeable constraints include: safe distance, communication distance, the final formation; the changeable constraints include: the motile threat sources and so on.

Step 5. Calculate $M_i(t)$ by using formula (3), (4) and (5), if abs($M_i(t)$) is far from □, we should calculate function diversity. By considering the value of diversity, which is calculated by formula (6), we can make sure whether to reset the particles.

Step 6. Use formula (1), (2) or (7), (8) to update the particles.

Step 7. If the iteration stops, the optimal solution is got. Therefore, the coordinates of multi-UAVs are calculated by formula (9).

The above procedure can also be illustrated with the following figure.

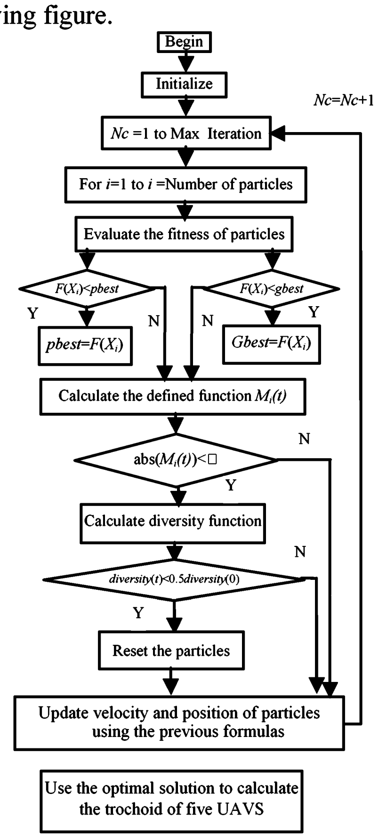

Fig. 1. Flowchart of the hybrid diversity PSO and time optima control in dynamic environments

VI. EXPERIMENTAL RESULTS

In order to investigate the feasibility and effectiveness of the PSO based on dynamic environment and the time optimal control to reconfiguration, series of experiments are conducted.

The final formation should meet : X_relative_1 = [-10,0,0] X_relative_2 = [10,0,0] X_relative_3 = [0,0,0] X_relative_4 = [0,-10,0] X_relative_5 = [0,10,0]. Choose the positions of the threat sources are as follows: [30, 10] in 1th~100th generation; [30, 15] in 101th~200th generation; [30, 20] in 201th~350th generation. The radius of threats and safe distance of two UAVs are both 5. Set the learning factors c_1=2, c_2=2, inertial weight w=0.9, number of particles M=80, dimension of solutions n=51, penalty factors are 1.2×10^7

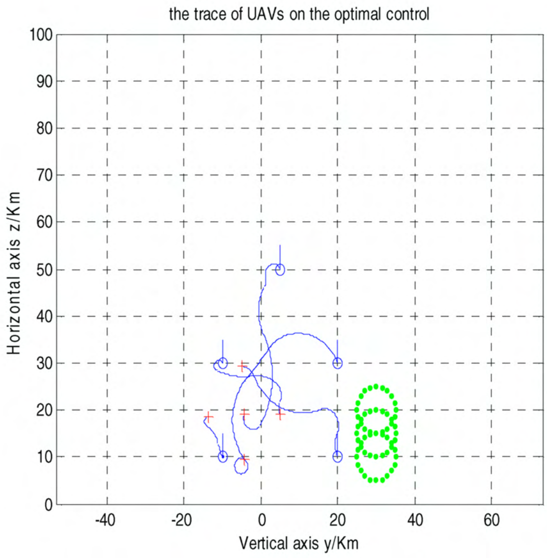

Fig. 2. Trochoid in diversity-PSO experiment (N_{cmax}=350)

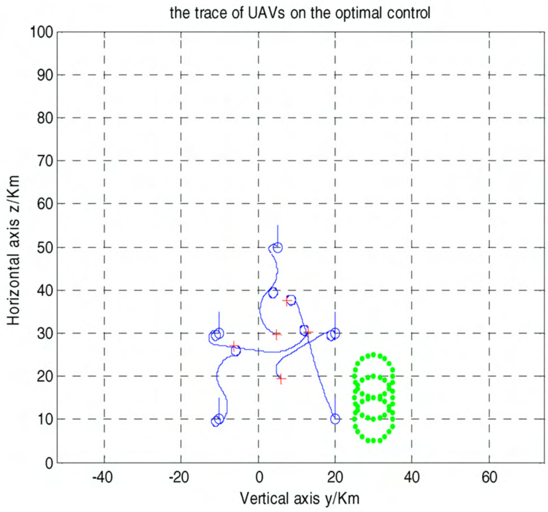

Fig. 3. Trochoid in PSO experiment (N_{cmax} =350)

Fig. 2 and Fig. 3 show that using diversity-PSO makes the final formation and the trochoid better than using PSO.

These two methods can both keep the UAVs away from the threat sources which we have set. But for PSO, the final formation is not good because of the influence of the changeable fitness. The formation using our hybrid diversity PSO and time optima control method is basically up to the standard. The trochoids of these two experiments aren't good because the number of iterations isn't enough, and the changeable fitness has a great influence on them.

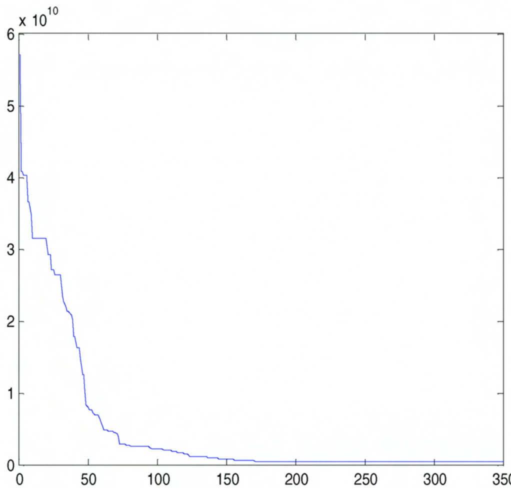

Fig. 4. Trend of the fitness in diversity-PSO experiment (Nc=350)

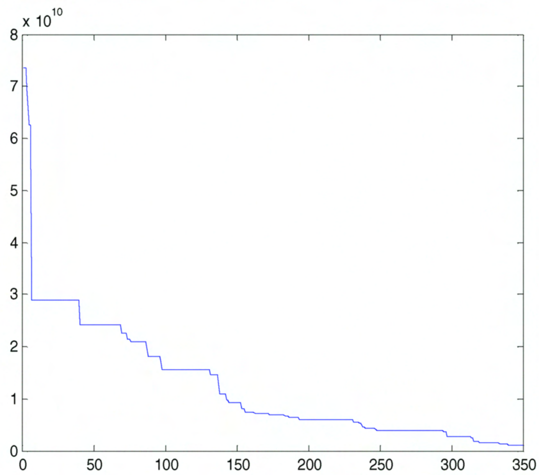

Fig. 5. Trend of the fitness in basic PSO experiment (N_{cmax} =350)

Fig.4 and Fig.5 show that the diversity-PSO can make the convergence of the fitness better.

The convergence of the fitness is a crucial criterion in judging the algorithms. The stair-like curve in fig. 5 demonstrates the adaptability of PSO to the dynamic environment is poor. The curve in fig. 4 demonstrates the good adaptability of diversity-PSO. Therefore, the response to dynamic and complicated environment can fulfill the requirements.

VII. CONCLUSION AND FUTURE WORK

This paper proposed a novel hybrid diversity PSO and time optima control approach to multi-UAVs formation reconfiguration in dynamic and complicated environments. Series simulation results are also presented to verify the feasibility and effectiveness of the proposed method.

Our future work will focus on how to describe the changes of the dynamic environments accurately, and how to establish a better mathematical model for multi-UAVs formation reconfiguration.

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